MODELLING LAVA FLOWS BY CELLULAR AUTOMATA and EXAMPLES OF SIMULATIONS OF ETNEAN EVENTS

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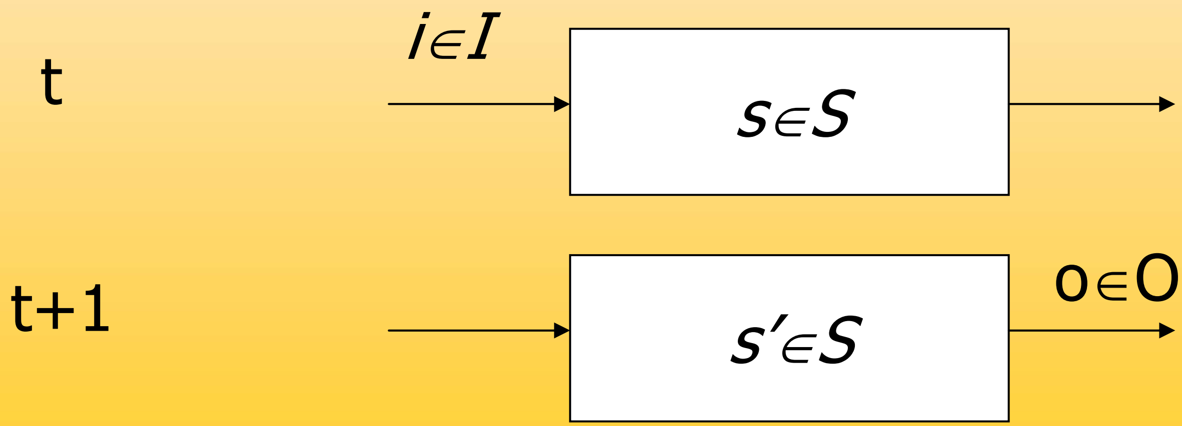
Finite Automaton

$$A = (I, S, O, \sigma, \delta)$$

I, ***S***, ***O*** finite sets of ***I***nput, ***S***tates and ***O***utput

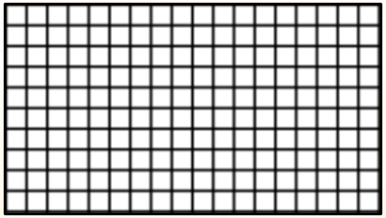
$\sigma : I \times S \dashrightarrow S$ state transition function

$\delta : I \times S \dashrightarrow O$ output function



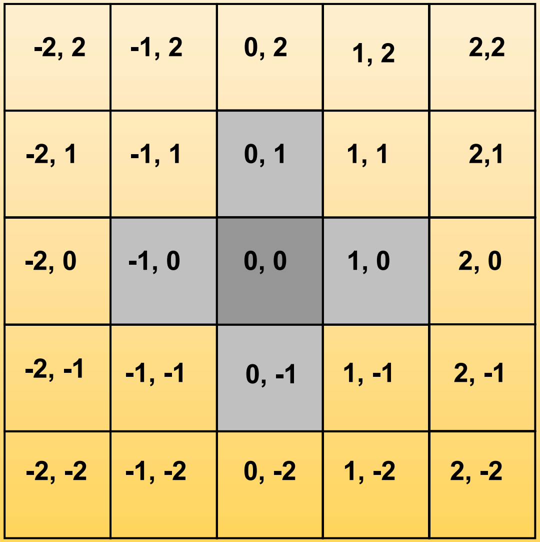
Cellular Automata (*CA*)

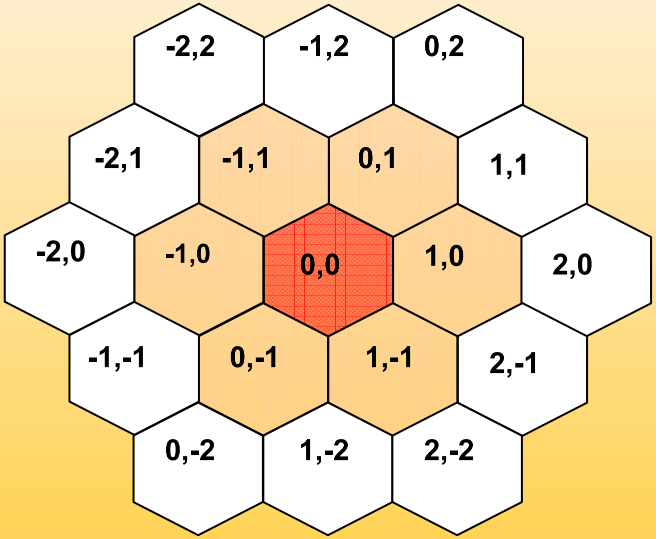
Intuitively a homogeneous Cellular Automaton **(*CA*)** can be seen as a d-dimensional space, partitioned in cells of uniform size, each one embedding an identical finite automaton, the elementary automaton **(*ea*).**



a fragment of a two-dimensional Cellular Automaton

Input for each cell is given by the states of the neighbouring cells, where the neighbourhood conditions are determined by a pattern invariant in time and space.





von Neumann neighbouring

hexagonal neighbouring

At the time *t=0*, cells are in arbitrary states and the *CA* evolves changing the state at discrete times, according to the transition function.

SOME *CA* CRITERIA FOR MODELLING MACROSCOPIC PHENOMENA

First requirement: the abstract CA must be related univocally to real phenomenon

- The cell corresponds usually to a portion of the space; so the cellular space must be three dimensional.
- Global parameters must be considered:
  at least the size of the cell and the time corresponding to the CA transition step;

These two parameters may effect the transition function implicitly.

SOME *CA* CRITERIA FOR MODELLING MACROSCOPIC PHENOMENA

Second requirement: the phenomenon macroscopicity needs compositeness of states and transition function

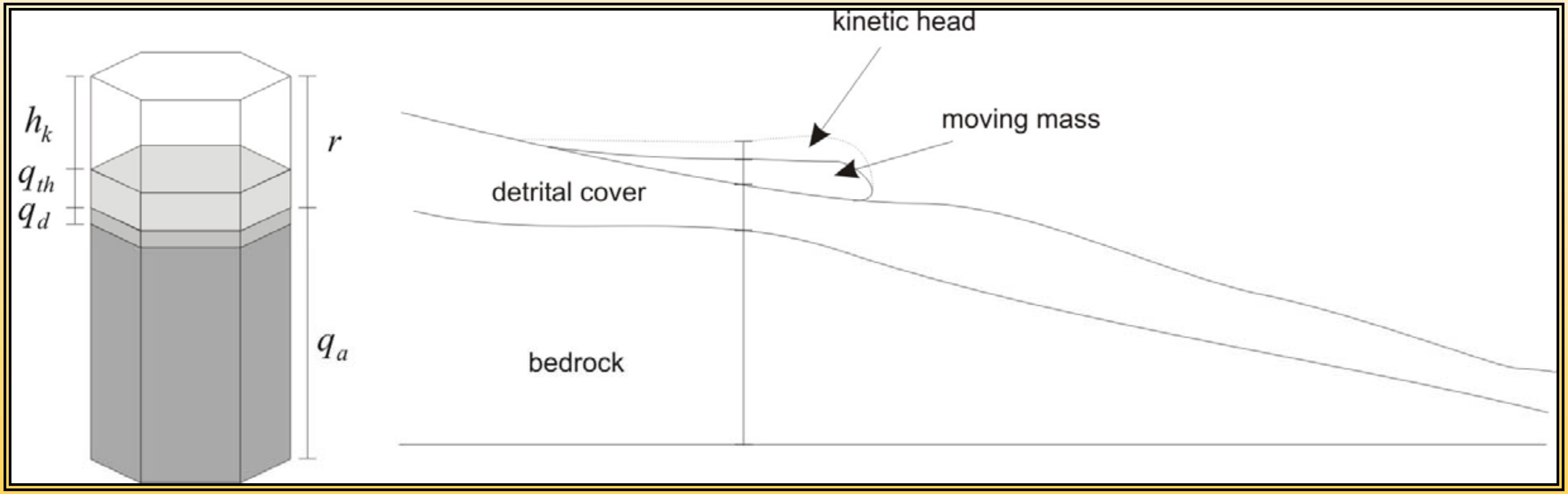
- Each characteristic, relevant to the evolution of the system and relative to the space portion corresponding to the cell, is individuated as a substate; the set ***Q*** of the states is given by the Cartesian product of the sets of substates: $Q = Q_1 \times Q_2 \times \ldots\ldots \times Q_n$ ***; the substates are constant in the space occupied by the cell (e.g. the temperature).***
- ***As the state of the cell can be decomposed in substates, the transition function may be split in many "elementary" processes: local interactions and internal transformations:***

<u>Local interactions</u> are changes due to interactions of substates in the neighbourhood.

<u>Internal transformations</u> are a borderline case of local interaction, defined as the changes in the values of the substates due to cell internal conditions (substates inside the cell),

A PRACTICAL APPROACH FOR MODELLING SURFACE FLOWS

First consideration: Two dimensions may be sufficient, because the third dimension may be regarded as a substate and inserted in the state of the cell (e.g. the altitude for phenomena concerning the earth surface).



A PRACTICAL APPROACH FOR MODELLING SURFACE FLOWS

Second consideration: The flows may be expressed as a substate, they must be managed in terms of local interactions and *must minimize locally unbalance conditions*. An algorithm for the differences minimisation have to be expressed in the context of discrete space and time and bounded by the cell neighbouring.

Third consideration: A relaxation rate, depending on both the cell size and the duration of the *CA* step, must be considered because the minimum "imbalance" conditions cannot be always achieved in a *CA* step. This mechanism involves particular care in the space and time settlement: the size of the cell limits at the top the *CA* step, because the outflow rate may not be so rapid that the outflow overcomes the neighbourhood boundaries in a step.

OUTFLOWS DETERMINATION BY THE MINIMISATION ALGORITHM

Local conditions for minimum unbalance conditions are determinant for the evolution of the system

Problem: Outflows from the central cell to the other n neighbouring cells must be determined in order to minimise the differences of a quantity q in the neighbouring cells:

Definitions

q_d = quantity, that may be distributed, in the central cell
q_0 = irremovable quantity in the central cell
q_i = quantity in the cell i $1 \leq i \leq n$
f_i = outflows from the central cell $0 \leq i \leq n$ (*f_0 is the part of q_d* remaining in the central cell)
$q_i' = q_i + f_i \quad 0 \leq i \leq n$
$q'min$ is the minimum value for q_i' $0 \leq i \leq n$

Bounds

$q_d = \Sigma_i f_i \quad 0 \leq i \leq n$
$\Sigma i\,(q_i' - q'min)$ *must be minimised by the values of* f_i
$(0 \leq i \leq n)$

OUTFLOWS DETERMINATION BY THE MINIMISATION ALGORITHM (continued)

Minimisation Algorithm

(a) All the neighbouring cells are "not eliminated": A is the set of not eliminated cells

(b) The "average q" (av_q) is found for the set A of not eliminated cells:

$$av\_q = (q_d + \Sigma_i\, q_i)/\#A \quad i \in A$$

(c) *The cell* x with $q_x > av\_q$ *is eliminated*

(d) *Go to step (c) until no cell is eliminated.*

(e) $f_i = av\_q - q_i \quad i \in A \qquad\qquad f_i = 0 \quad i \notin A$

Relaxation rate r

The relaxation rate accounts for the determination of the part of flow that is effectively transferred in the neighbour cell from the central cell in a CA step.

Cellular Automata

	1	
3	**0**	**2**
	4	

EXAMPLE OF DISTRIBUTION (von Neumann neighbouring):

$$q_d = 9,\quad q_0 = 81,\quad q_1 = 100,\quad q_2 = 76,\quad q_3 = 83,\quad q_4 = 71,$$

	100	
83	81:**9**	76
	71	

av_h=420/5=84
cell 1 is eliminated

	*	
83	81:**9**	76
	71	

av_h=320/4=80
cells 0 and 3 are eliminated

	*	
*	***9**	76
	71	

av_h=156/2=78
no cell is eliminated

	100	
83	81	76:**2**
	71:**7**	

$f_2=2$ $f_4=7$

$f_0=f_1=f_3=0$

CA and CA-like models for lava flow simulation

- Barca, Crisci, Di Gregorio, Nicoletta (1986-89).

  First approach to the Cellular Automata Model. Three and two dimensional models with discrete time and space, allowing multiple flows;

- Ishihara et al. (1988, 1989).

  Starting from Navier-Stokes equations, adopting a space tessellation and discrete time. However, not applicable to multi-flows and/or extruded intermittently flows;

- Young and Wadge (1990).

  Cellular Automata approach with simulation of simple advancing lava flow fronts (FLOWFRONT);

- Crisci, Di Gregorio, Rongo, Spataro (1990-2002)

  Cellular Automata Model with the minimisation algorithm (SCIARA).

- Miyamoto H. and Sasaki S. (1997)

  Simulating lava flows by an improved cellular automata method. They solved the problem of spurious symmetries with a probabilistic method

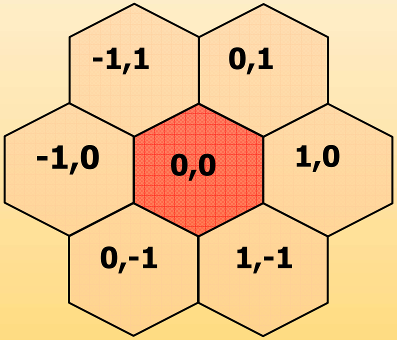
SCIARA: S*imulation by* **C***ellular* **I***nteractive* **A***utomata of the* **R***heology of* **A***etnean lava flows* (release hex1)

SCIARA = $\langle \mathbf{R}, \mathbf{X}, \mathbf{L}, \mathbf{Q}, \mathbf{P}, \sigma, \gamma \rangle$

- **R** = $\{(x, y) \mid x, y \in N, 0 \le x \le lx, 0 \le y \le ly\}$ **is the set of points with integer co-ordinates in the finite region, where the phenomenon evolves.** *N* is the set of natural numbers.

- $\mathbf{L} \subset \mathbf{R}$ specifies the lava source cells

- **X** = {(0,0), (0,1), (0,1), (1,0), (-1,0), (-1,1), (1,-1)} is the set, which identifies the geometrical pattern of the cells, which influence the cell state change.



The finite set **Q** of states of the ea: $\mathbf{Q} = \mathbf{Q_a} \times \mathbf{Q_{th}} \times \mathbf{Q_T} \times \mathbf{Q_o}^6 \times \mathbf{Q_i}^6$

Q_a	altitude of the cell
Q_{th}	lava thickness in the cell
Q_T	lava temperature in the cell
Q_o (Q_i)	lava outflow (inflow)

SCIARA Parameters

SCIARA (release hex1)

- **P** is the set of *global parameters* of SCIARA

$$\mathbf{P} = \{p_c, p_t, p_{adh\_v}, p_{adh\_s}, p_{Tv}, p_{Ts}, p_r, p_c\}$$

p_c	side of the cell	5 m
p_t	temporal correspondence of a step of SCIARA	60 s
p_{adh_v}	lava adhesion at the vents	0.7 m
p_{adh_s}	lava adhesion at the solidification	10 m
p_{Tv}	lava temperature at the vents	1373 K
p_{Ts}	lava temperature at solidification	1123 K
p_r	relaxation rate	1
p_c	cooling parameter	$1.4\ 10^{-14}\ (m/K)^3$

SCIARA Transition Function

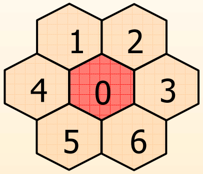
- **σ: $Q^7 \to Q$** is the deterministic transition function of the ***CA***

elementary process in order	*input*	*variations/determinations*	*updating*
solidification	$Q_{th} \times Q_a \times Q_T$	$\Delta(Q_a , Q_{th})$	Q_a , Q_{th}
lava outflows	$(Q_a \times Q_{th})^7 \times Q_T$	$Q_o^{\ 6}$	$Q_o^{\ 6} , Q_i^{\ 6}$
lava mixing	$(Q_i \times Q_o)^6 \times Q_{th} \times Q_T^{\ 7}$	Q_{th}, Q_T	Q_{th}, Q_T
lava cooling	$Q_{th} \times Q_T$	Q_T	Q_T

- **γ:L$\times$*N*$\to$ Q_{th} specifies the emitted lava from the source cell at the step s, (s$\in$*N*)**

SCIARA Transition Function

SCIARA (release hex1) **transition function σ**



Internal transformation **SOLIDIFICATION** $\sigma_S: Q_{th} \times Q_a \times Q_T \rightarrow Q_a \times Q_{th}$

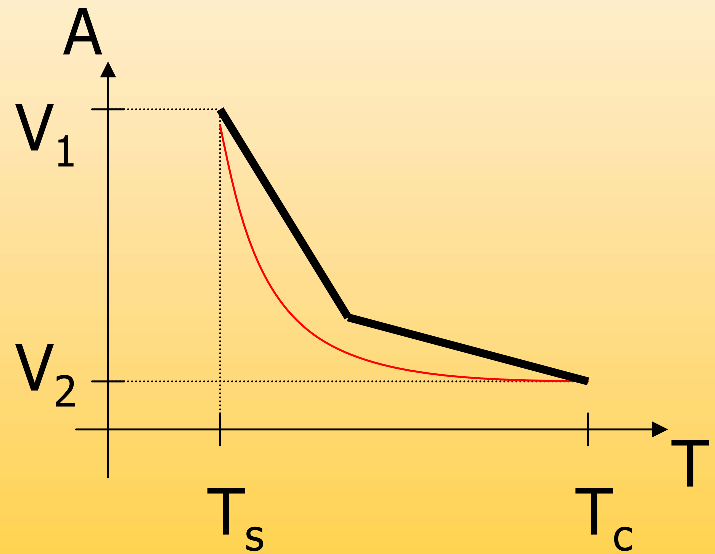
The cell altitude remains unchanged until solidification condition holds: ($Q_T < p_{Ts}$), then the altitude is increased by the lava thickness and lava thickness is zeroed.

SCIARA Transition Function

Local interaction: LAVA OUTFLOWS $\sigma_{LO}: (Q_{th} \times Q_a)^7 \times Q_T \rightarrow Q_o^6$

■**Lava's rheological resistance is strongly dependent on temperature and the resistance increases as the temperature decreases. Due to difficulties in specifying lava rheology and and its variation with temperature, we use an adherence parameter v that represents the amount of lava that *remains* in each cell at each step.**

$$v = a\, e^{-bT}$$



SCIARA Transition Function

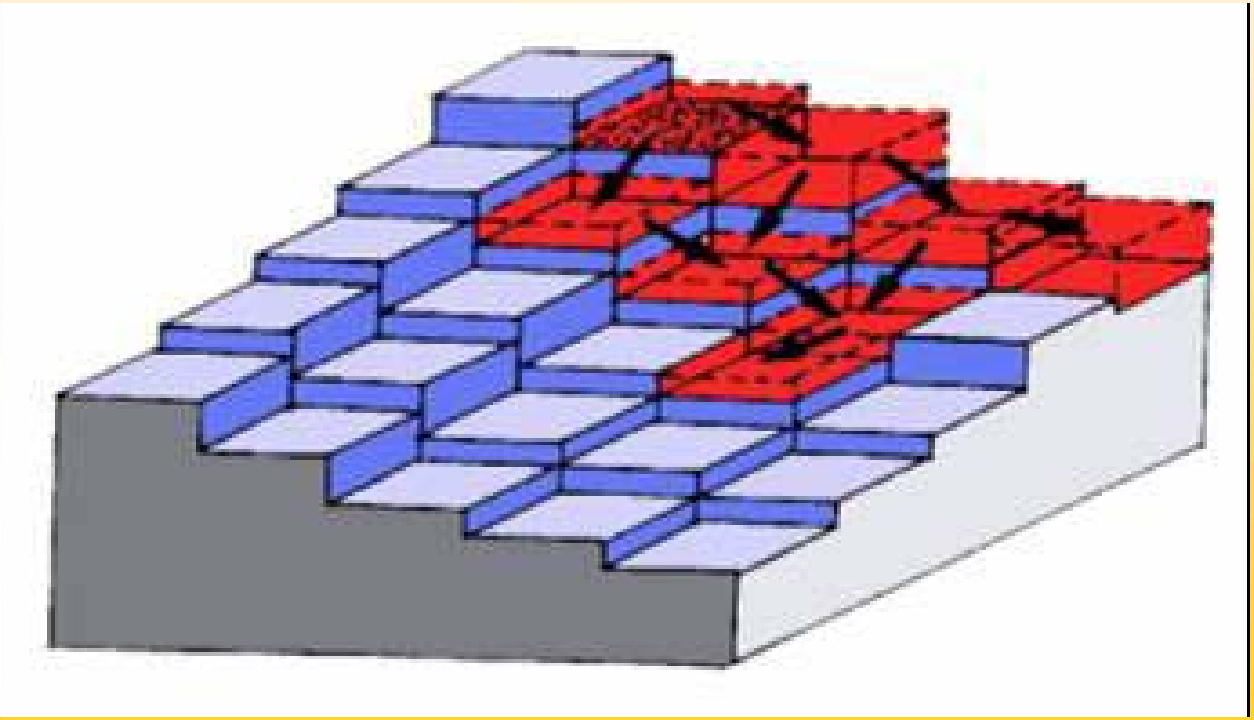
Local interaction: LAVA OUTFLOWS $\sigma_{LO}: (Q_{th} \times Q_a)^7 \times Q_T \rightarrow Q_o^6$

Minimisation algorithm application

q_d = quantity, that may be distributed, in the central cell = $\mathbf{Q_{th}}$ **- adhesion**

q_0 = irremovable quantity in the central cell = $\mathbf{Q_a}$ **+ adhesion**

q_i = quantity in the cell i $\mathbf{1 \leq i \leq 6 = Q_a + Q_{th}}$



SCIARA Transition Function

Local interaction: LAVA MIXING $\sigma_{LM}: Q_{th} \times Q_i^6 \times Q_o^6 \times Q_r^7 \rightarrow Q_r$

Lava mixing involves the determination for the central cell:

a) the remaining lava thickness (**rem_th**): $\mathbf{rem\_th = Q_{th}[0] - \Sigma_j\ Q_O[j]}$ $\mathbf{1 \le j \le 6}$

b) new lava thickness (**new_th**): $\mathbf{new\_th = rem\_th + \Sigma_j\ Q_i[j]}$ $\mathbf{1 \le j \le 6}$

c) the temperature variation by mixing is calculated as the average weight of Q_T, by considering both the remaining lava and the inflows:

$$\mathbf{new\_T = (rem\_th * Q_T[0]) + \Sigma_j (Q_i[j] * Q_T[j])) / (rem\_th + \Sigma_j\ Q_i[j])}$$

$\mathbf{1 \le j \le 6}$

SCIARA Transition Function

Internal transformation LAVA COOLING $\sigma_{LC}: Q_{th} \times Q_T \rightarrow Q_T$

Temperature drop due to irradiation at the surface is computed, assuming that other losses are not relevant:

$$new\_T = Q_T \Big/ \sqrt[3]{1 + (Q_T^3 \cdot p_c / Q_{th})}$$

from

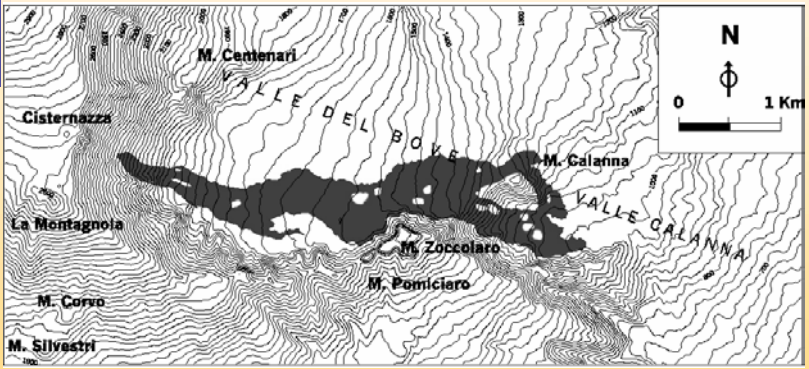
$$T = T_{av} / \sqrt[3]{1 + (3T_{av}^3 \varepsilon\sigma A \Delta t)/(\rho c V)} = T_{av} / \sqrt[3]{1 + (T_{av}^3 pA / V)}$$

where ρ is the lava density, c the specific heat, V the volume, σ the Stephan-Boltzmann constant, T the absolute temperature of the surface, A the surface area of the cell, ε is the surface emissivity, Δt ($\Delta t = t2 - t1$), the time interval, is the step of the CA, $p=3\varepsilon\sigma\Delta t/\rho c$ is the "cooling parameter", with $3\sigma\varepsilon/\rho c$ describing the lava's physical properties. Moreover, Δt is dependent also on the cell side dimension.

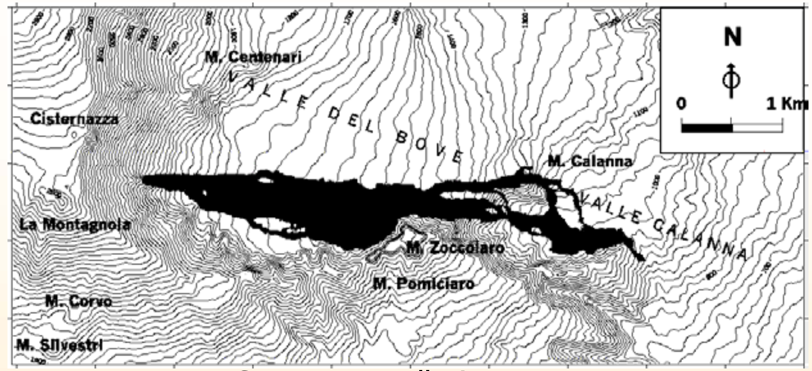
Model validation

- Model validation

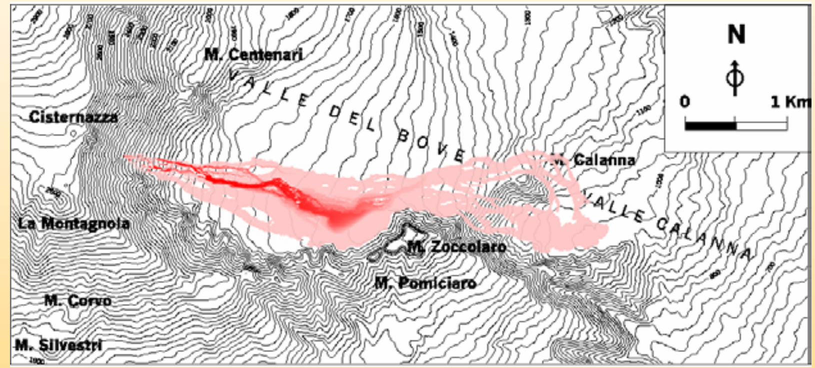
(Valle del Bove, Mt Etna, 1991)



Real event



Square tessellation



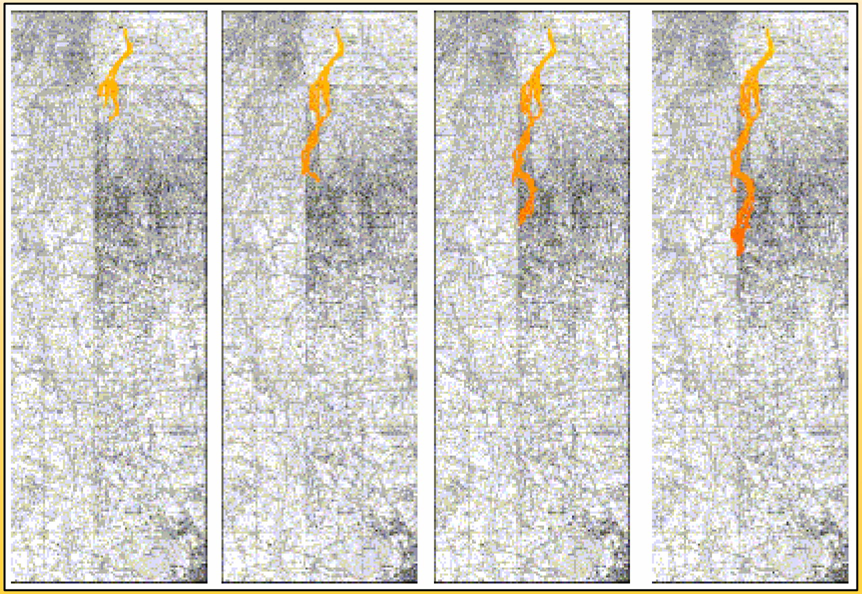
Hexagonal tesselation

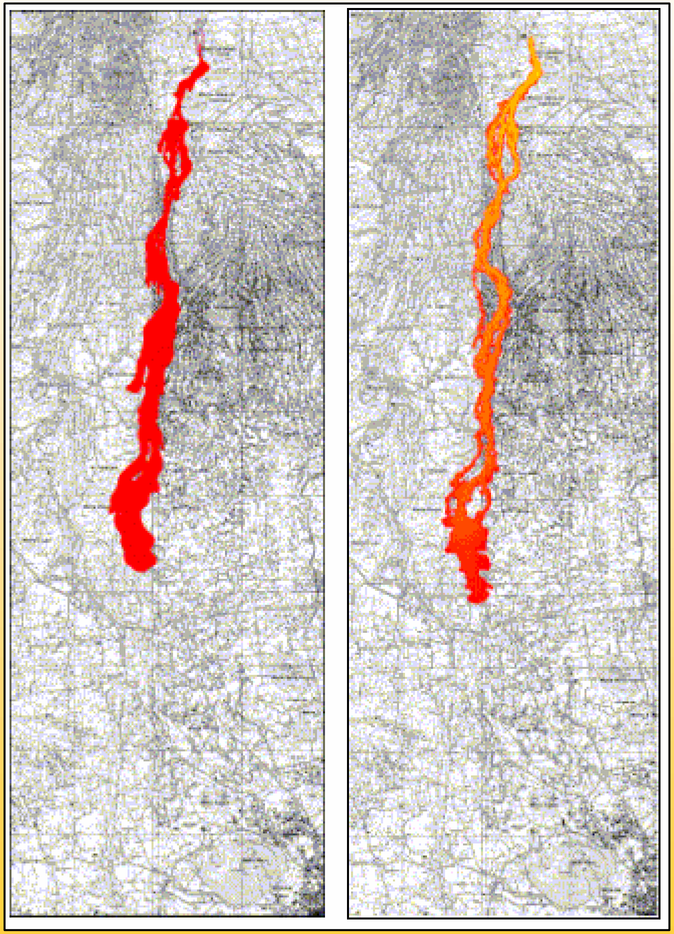
New explicit velocity model

SCIARA Real Application

SCIARA (release hex1): simulation of the July 2001 Etnean eruption

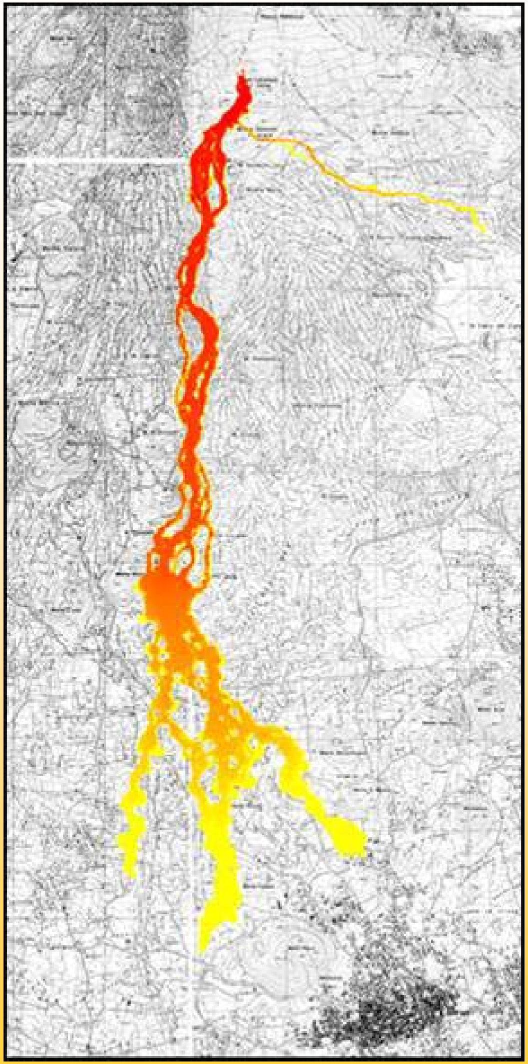
The Etnean eruption started the 18th of July 2001 at an elevation of ca.2100 m, near Nicolosi (Sicily). The lava threatened Nicolosi, however stopping after 10 days (maximum lava field length).
Simulation with SCIARA were carried to develop future scenarios and/or possible human intervention.





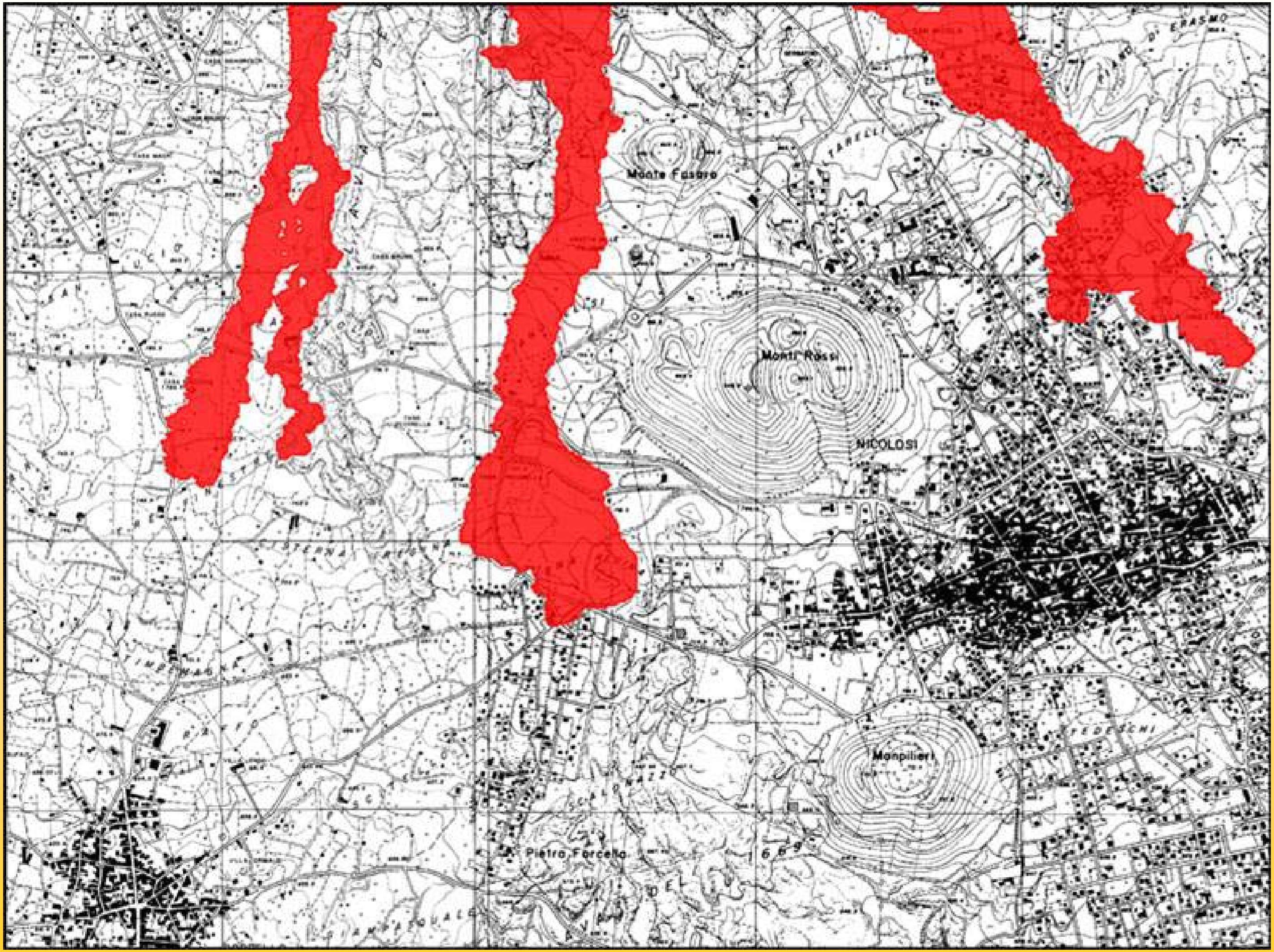
Scenario of 2001 Etnean event

A possible future scenario after 40 days at 12 m^3/s



Scenario of 2001 Etnean event

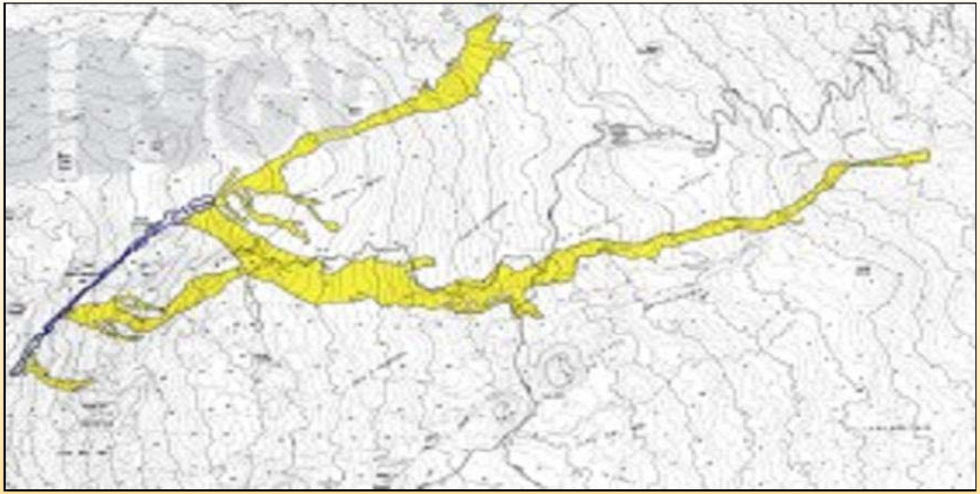
A possible future scenario for Nicolosi after 100 days at 24 m^3/s



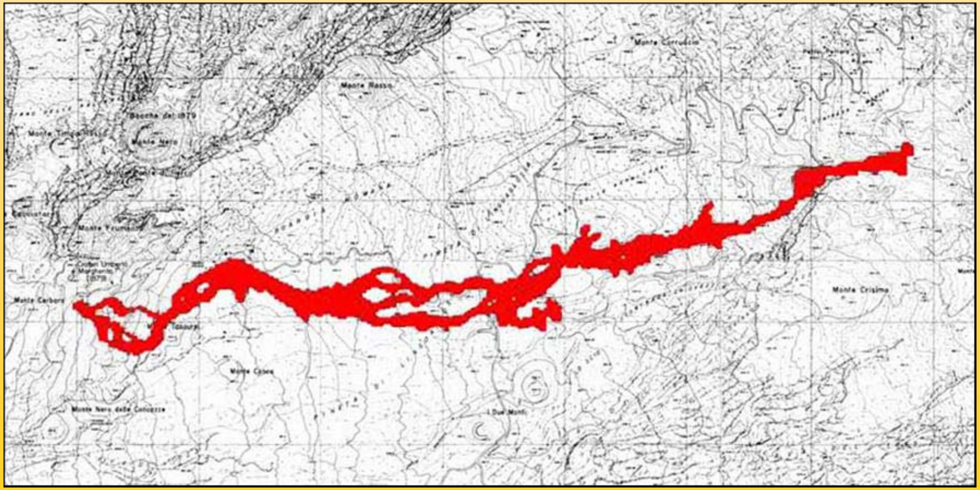
2002 Etnean eruption simulation

Real vs simulated event in 2002 Etnean eruption (Linguaglossa)

Real event



Simulation with only a vent



Comments and Conclusions (1)

- Cellular Automata represent an alternative approach to differential equations in modelling complex systems, whose evolution is strongly dependent on local interactions of their constituent parts.

- The empirical method, here introduced, was successfully applied by the research group “Empedocles” to other macroscopic complex phenomena, such as soil contamination and bioremediation, forest fires, soil erosion by rain; new application fields are considered: pyroclastic flows, marine environment evolution.

- This empirical method permits to start with simple models, whose refinement can be performed in an incremental way, introducing other internal transformations and local interactions. This allows a careful monitoring of the model building phase by comparison between real phenomena and simulations.

- This empirical method involves, for each internal transformation or local interaction, the introduction of problem-specific parameters, whose determination may be performed by applying optimization methods to minimize the difference between model results and experimental data. Genetic algorithms were effective for applications with several parameters.

Comments and Conclusions (2)

- It is important to define the limits to the application of the model to similar phenomena: e.g., SCIARA was validated for the Etnean lava flows in 1986/7 eruption and 1991/2 eruption. SCIARA application during the eruption in the 2001 Summer for the hazard analysis was possible, because Etnean lavas features don’t change significantly in the time. Cases, where the features change, involve a validation considering an interval of possible values of parameters, corresponding to different typologies of cases.

- This point is crucial; investigantions showed that there are different confidence intervals for phenomena of the same type.

- A last consideration can be added: the decomposition of the complex macroscopic phenomenon in internal transformations and local interactions seems to have encouraged interdisciplinary cooperations and exchange of information, at least in the case treated here.