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Constrained Coalition Formation on Valuation Structures

Gianluigi Greco and Antonella Guzzo

University of Calabria, Italy

UNIVERSITÀ DELLA CALABRIA

Transferable Utility Games

- A transferable utility game is a pair (N, v), where:
  - $N = \{a_1, ..., a_n\}$ is the set of agents
  - $v: 2^N \to \mathbb{R}$ is the characteristic function
    - for each subset of players C, $v(C)$ is the amount that the members of C can earn by working together

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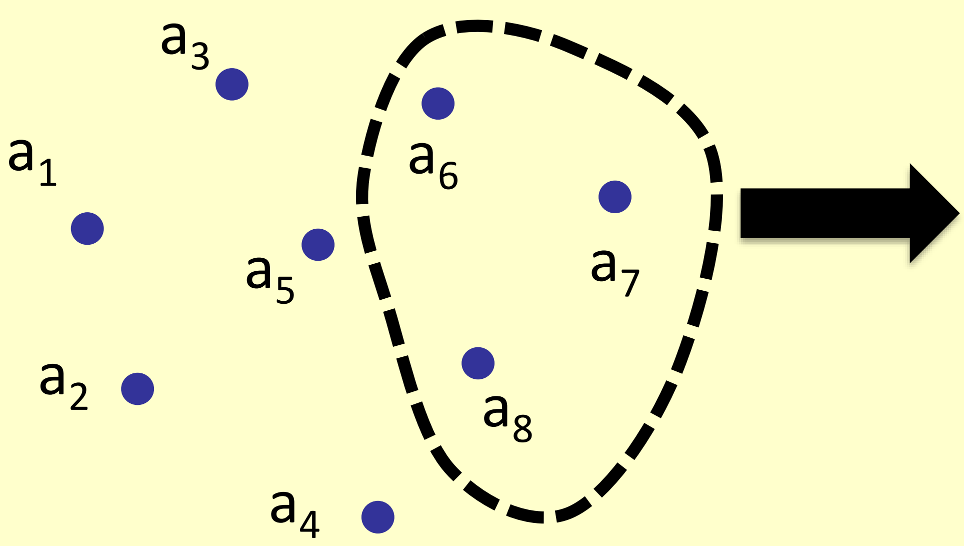

$v(\{a_6,a_7,a_8\})$ = **15$**

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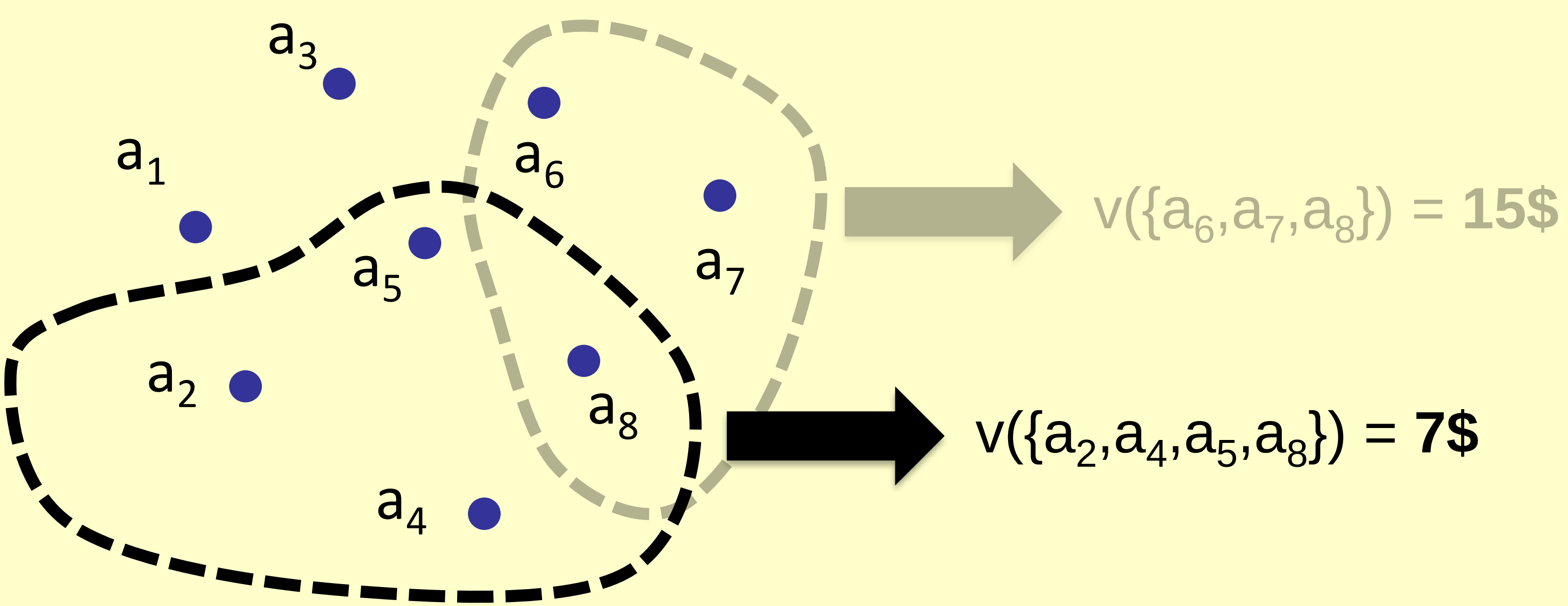
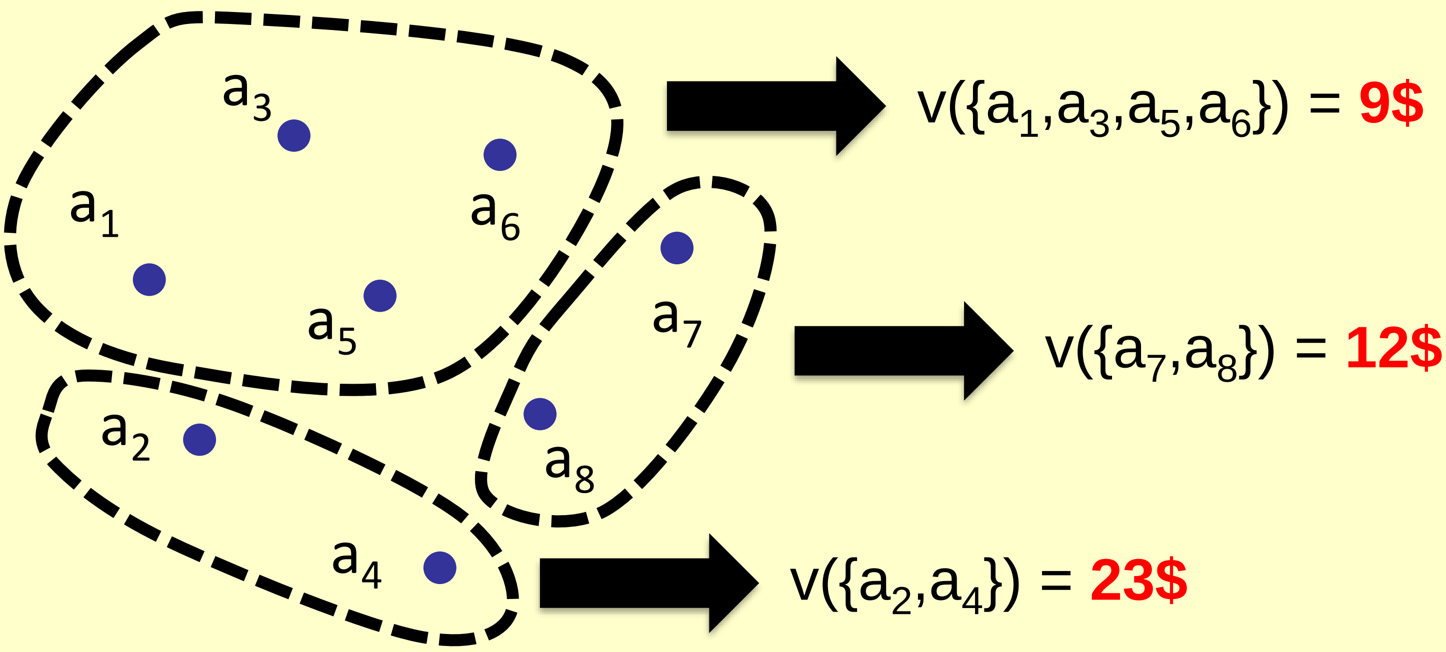


Coalition Structures

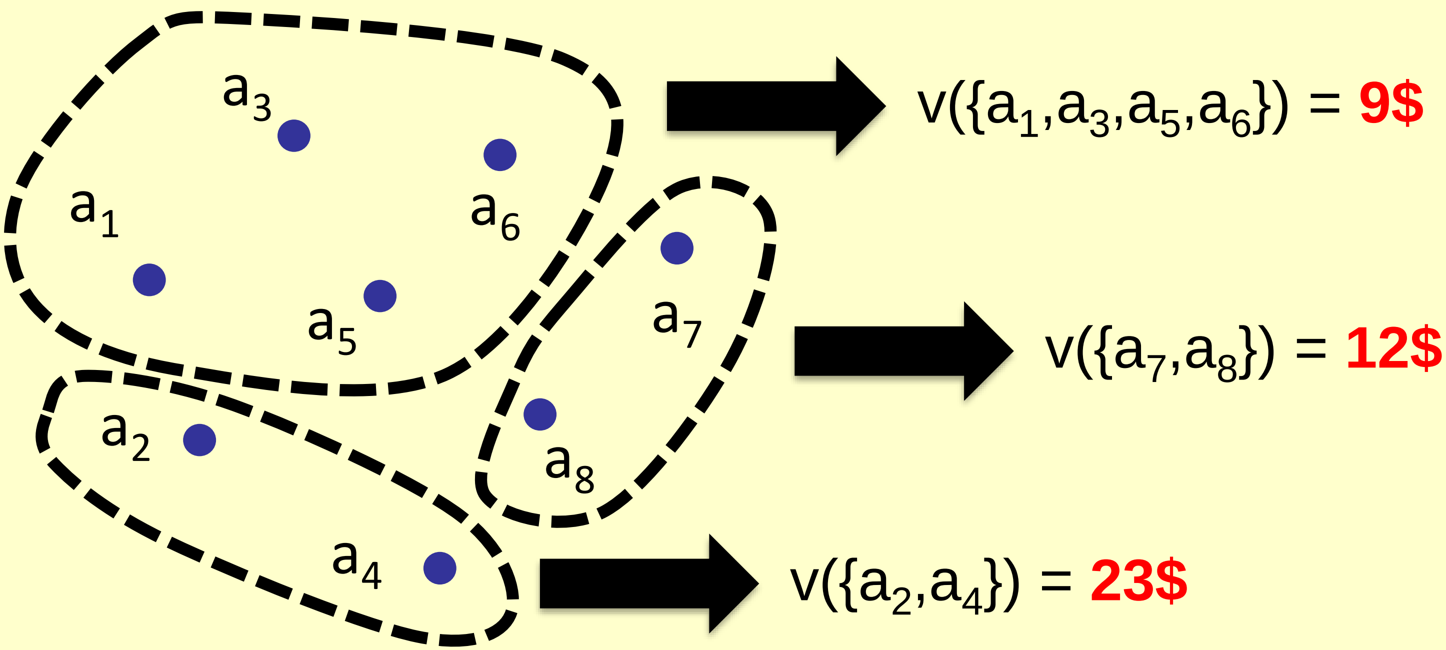
- A partition of the agents in exaustive and disjoint coalitions
  - Every agent belongs to some coalition
  - Coalitions do not overlap
- The value is the sum of the values of the coalitions



Problem of Interest

- **Input**: A coalitional game
- **Ouput**: The "optimal coalition structure"…

  …that is, the structure with the greatest overall value



Outline

- **Constraints on Coalition Formation**
- Islands of Tractability
- From Optimality to Stability

Constraints on Coalition Structures

- In the real world, some coalition structures might be not admissible, because they violate constraints induced by the specific semantics of the applications at hand

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- Constraints on the interactions (e.g., physical limitations)
  [G. Demange, 2009]
- Size of coalitions
  [O. Shehory and S. Kraus, 1998]
  [T. Rahwan and N. R. Jennings, 2007]
- Positive and negative constraints (via a suitable language)
  [T. Rahwan, T. P. Michalak, E. Elkind, P. Faliszewski, J. Sroka, M. Wooldridge, and N. R. Jennings, 2011]

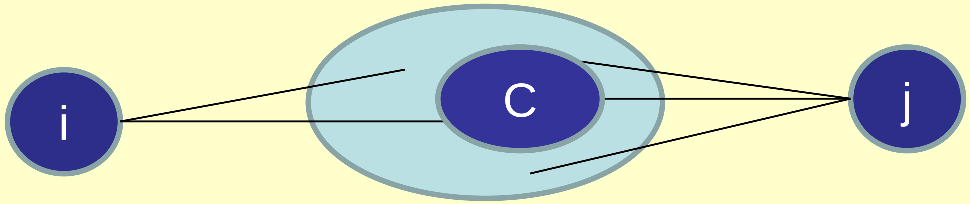
«Constraints» on Worth Functions

- Even the worth function can be subject to constraints

«Constraints» on Worth Functions

- Even the worth function can be subject to constraints
  - Independent on Disconnected Members (IDM)
    [T. Voice, M. Polukarov, and N. R. Jennings, 2012]

An interaction graph is given, and any two agents have no effect on each other’s marginal contribution to their separator



- **i** and **j** are not directly connected
- For each coalition C that does not include i or j, it holds that

$$v(\mathbf{C} \cup \{\mathbf{i}, \mathbf{j}\}) = v(\mathbf{C} \cup \{\mathbf{i}\}) + v(\mathbf{C} \cup \{\mathbf{j}\}) - v(\mathbf{C})$$

Valuation Structures

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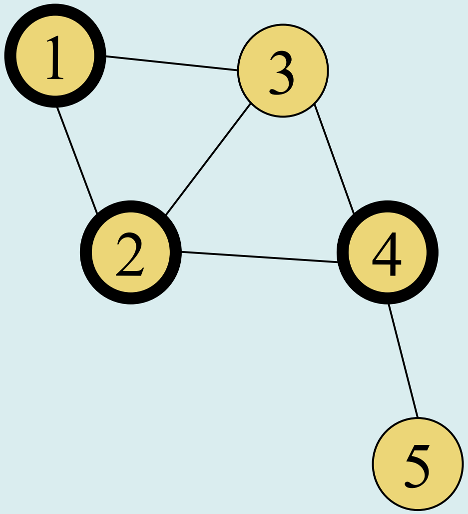
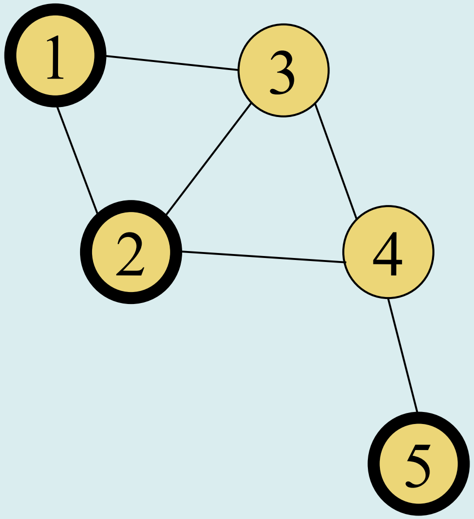
G is the interaction graph

- A coalition C is considered as a *feasible* one, only if the subgraph induced over the nodes in C is connected





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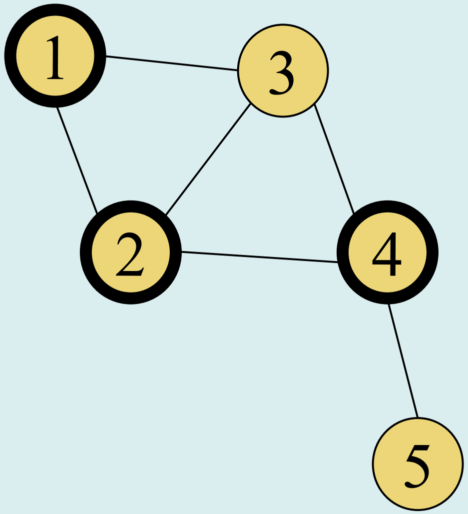

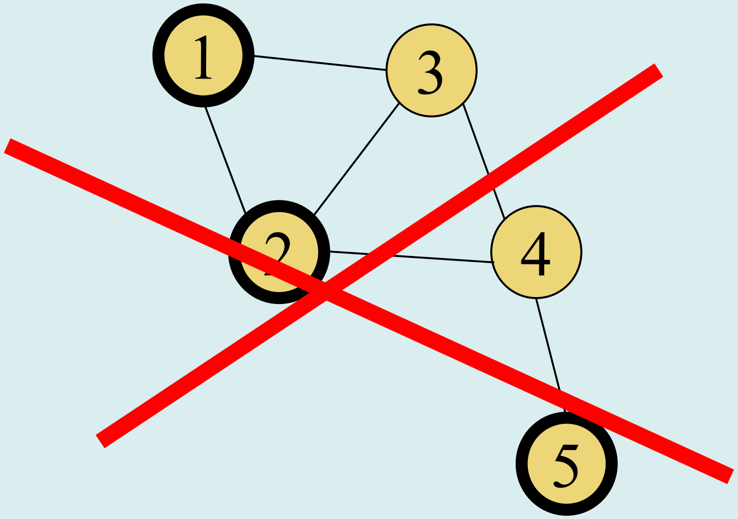

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_S_ is a set of pivotal agents

- They are pairwise "incompatible", so that every coalition C must also satisfy the condition $|S \cap C| \leq 1$ in order to be a feasible one

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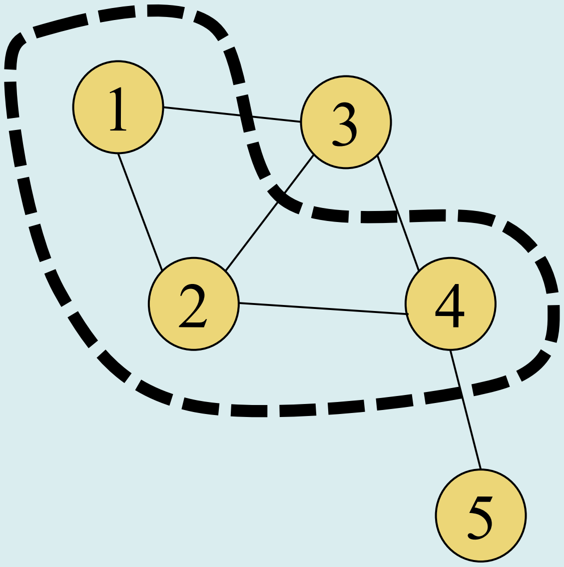

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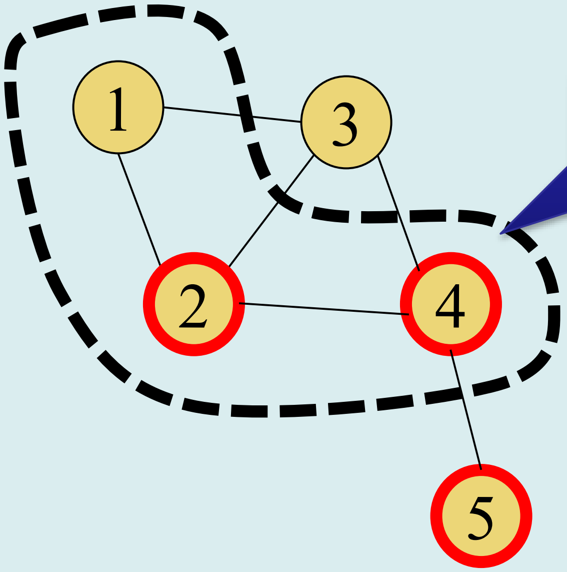

If {2,4,5} is the set of pivotal agents, then {1,2,4} is no longer a feasible coalition

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- We start by focusing on IDM functions…
- …but we allow a generalization to «affine transformtions»

$$\alpha, \beta : S \mapsto \mathbb{Q} \qquad x, y \in \mathbb{Q}$$

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$$val_\sigma(v, C) = \begin{cases} \alpha(a_i) \times v(C) + \beta(a_i) & \text{if } \{a_i\} = C \cap S, \\ x \times v(C) + y & \text{if } C \cap S = \emptyset \end{cases}$$

IDM function

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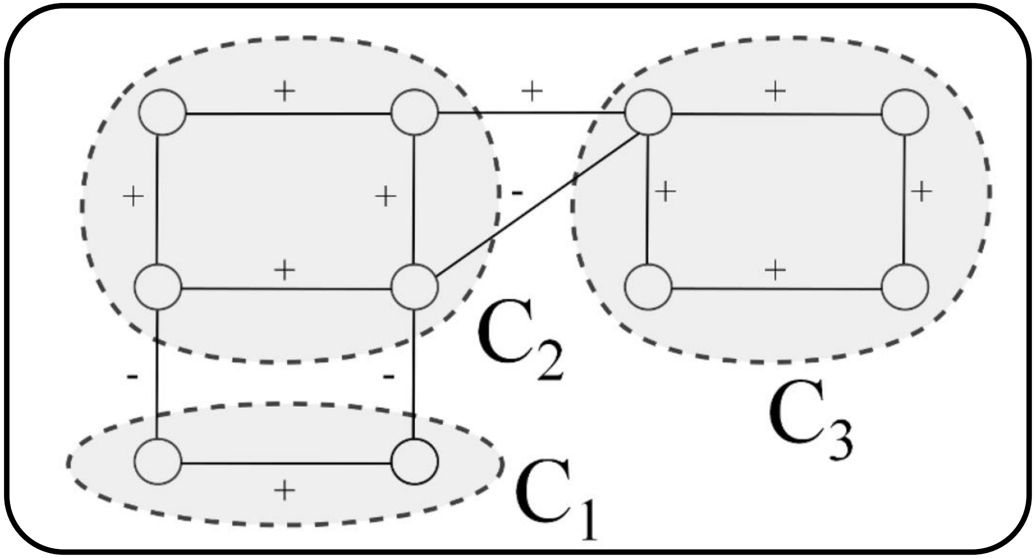
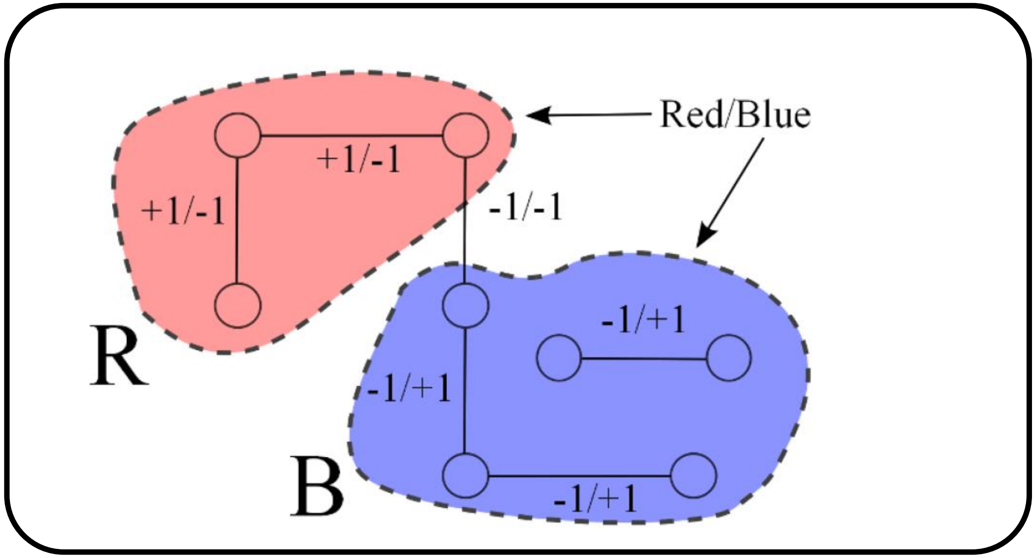
IDM function

In general, this is no longer an IDM one!

Clustering Problems

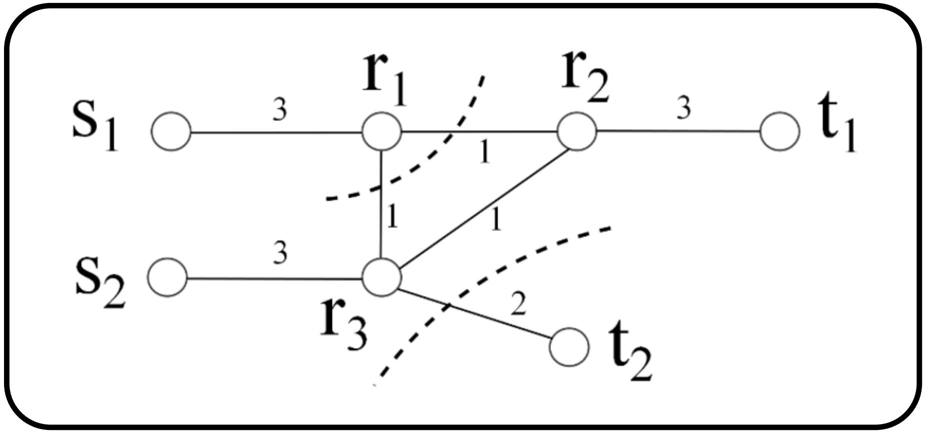


- In the ***k*-**correlation clustering, the value of a clustering is the number of + edges within the ***k*** clusters plus the number of – edges among clusters
- Find a ***k*-**clustering with maximum weight

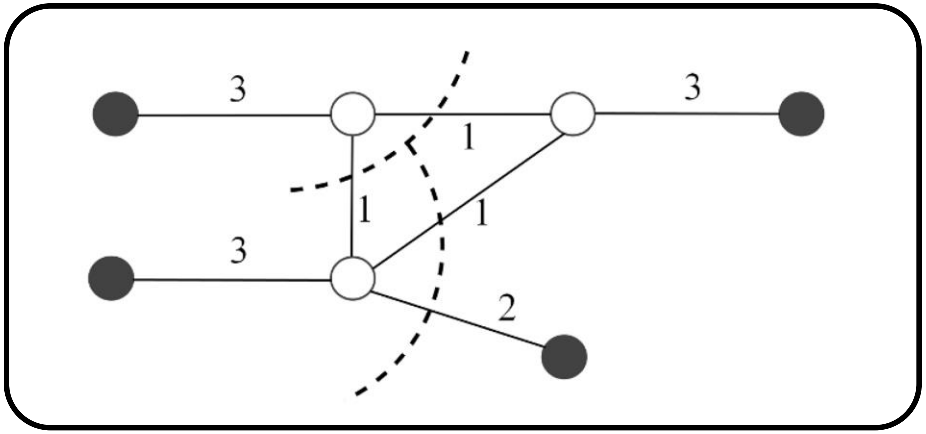


- In the chromatic clustering, the value of a clustering is the number of the weights of the edges within the clusters
- Weigths depend on the color assigned to the cluster
- Find a clustering with maximum weigth

Cut Problems



- A multicut is a set of edges separating all source/terminal pairs: s_1/t_1, s_2/t_2, …
- Find a multicut whose edges have minimum total weight



- A multiway cut is a set of edges separating all pair of terminals fro each other
- Find a multiway cut whose edges have minimum total weight

Outline

Constraints on Coalition Formation

Islands of Tractability

From Optimality to Stability

Related Result in the Literature

Theorem: *Coalition structure generation is tractable over IDM functions defined over interaction graphs that are* ***nearly-****acyclic (****bounded treewidth****).*

[T. Voice, M. Polukarov, and N. R. Jennings, 2012]

Our Main Result

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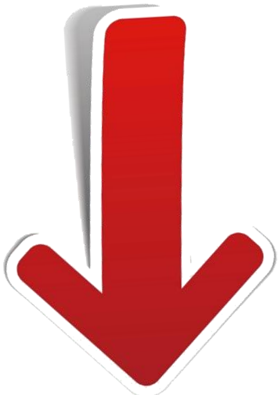

Theorem: *Coalition structure generation is tractable over* ***valuation structures*** *defined over interaction graphs that are* ***nearly-****acyclic.*

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- **Pivotal agents**
- **Affine trasformations from IDM functions**

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- **Pivotal agents**
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valuation structures and MC-nets

CSP encodings for MC-nets

novel machineries to encode connectivity

Corollaries

- The following problems are tractable when restricted over graphs having bounded treewidth:
  - *k*-clustering
  - chromatic clustering
  - multicut
  - multiway cut

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Transferable Utility Games: Stability

The core of a game is the set of all stable outcomes, that is, no coalition wants to deviate from

$$core(G) = \{(CS, \underline{\mathbf{x}}) \mid \Sigma_{i \in C}\, x_i \geq v(C) \text{ for any } C \subseteq N\}$$

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coalition structure

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worth distribution over the agents

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stability condition

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$val_\sigma(v, C)$

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Summary of Results

Theorem: *Computing the core is intractable.*

What happens with valuation structures?

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What happens with valuation structures?

Theorem: *The coalition structure core can be computed in polynomial time on valuation structures defined over interaction graphs that are nearly-acyclic.*

Thank you!