



GRAMMATICHE LL(1)

1. Calcolare FIRST e FOLLOW dei simboli non terminali della seguente grammatica:

$P \rightarrow \text{begin } L \text{ end}$
 $L \rightarrow ST$
 $T \rightarrow ST \mid \varepsilon$
 $S \rightarrow \text{id} := E; \mid \text{read}(\text{id}); \mid \text{write}(E);$
 $E \rightarrow FG$
 $G \rightarrow + FG \mid \varepsilon$
 $F \rightarrow (E) \mid \text{id}$

Calcolare gli insiemi guida della grammatica dell'esercizio precedente e dire se la grammatica è LL(1), motivando la risposta.

2. Sia data la seguente grammatica:

$S \rightarrow a A c S \mid c$	$R \rightarrow k C k \mid h C$
$A \rightarrow c P e \mid A e c$	$X \rightarrow X c \mid c a$
$P \rightarrow c X \mid c H c$	$C \rightarrow i A V \mid R$
$H \rightarrow H a \mid \varepsilon$	$V \rightarrow V e \mid V A V \mid \varepsilon$

- (a) Verificare se essa genera un linguaggio di tipo LL(1);
(b) Realizzare la tabella di parsing di un automa (anche non deterministico nel caso in cui il linguaggio generato non sia LL(1)) che contenga almeno la riga corrispondente ai non terminali: S e A.

3. Sia data la seguente grammatica:

$S \rightarrow h A b S \mid c$
 $D \rightarrow g E \mid g H d$
 $A \rightarrow b D D e \mid D f$
 $E \rightarrow E t \mid w$
 $B \rightarrow C k \mid h$
 $H \rightarrow H w H \mid H H w \mid i$
 $C \rightarrow i A D \mid B$
 $Z \rightarrow D Z \mid E$

- (a) Verificare se essa genera un linguaggio di tipo LL(1);
- (b) Realizzare la tabella di parsing di un automa (anche non deterministico nel caso in cui il linguaggio generato non sia LL(1)) che contenga almeno la riga corrispondente ai non terminali: S e E.



SOLUZIONI

1.

FIRST (S) = { id, read, write }
FIRST (F) = { (, id }
FIRST (G) = { +, ε }
FIRST (T) = { id, read, write, ε }
FIRST (E) = { (, id }
FIRST (L) = { id, read, write }
FIRST (P) = { begin }

FOLLOW (P) = { \$ }
FOLLOW (L) = { end }
FOLLOW (E) = { ;,) }
FOLLOW (T) = { end }
FOLLOW (G) = { ;,) }
FOLLOW (S) = { id, read, write, end }
FOLLOW (F) = { +, ;,) }

IG (P → begin L end) = { begin }
IG (L → ST) = { id, read, write }
IG (T → ST) = { id, read, write }
IG (T → ε) = { end }
IG (S → id := E;) = { id }
IG (S → read (id) ;) = { read }
IG (S → write (E) ;) = { write }
IG (E → FG) = { (, id }
IG (G → + FG) = { + }
IG (G → ε) = { ;,) }
IG (F → (E)) = { (}
IG (F → id) = { id }

La grammatica e' LL(1) perche' gli insiemi guida delle produzioni per lo stesso non terminale hanno intersezione vuota.

Costruiamo, come prova, la tabella di parsing:

	begin	end	id	read	write	()	;	+	\$
P	→begin L end									
L			→ST	→ST	→ST					
T		→ ε	→ST	→ST	→ST					
S			→ □ id := E;	→ read (id) ;	→ write (E) ;					
E			→ □ FG			→ □ F				
G							→ ε	→ ε	→ + FG	

F			→ id			→ □(E)				
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2.

Eliminiamo la ricorsione sinistra

$$\begin{aligned}
 S &\rightarrow a A c S \mid c \\
 A &\rightarrow c P e A' \\
 A' &\rightarrow e c A' \mid \varepsilon \\
 P &\rightarrow c X \mid c H c \\
 H &\rightarrow H' \\
 H' &\rightarrow a H' \mid \varepsilon \\
 R &\rightarrow k C k \mid h C \\
 X &\rightarrow c a X' \\
 X' &\rightarrow c X' \mid \varepsilon \\
 C &\rightarrow i A V \mid R \\
 V &\rightarrow V' \\
 V' &\rightarrow e V' \mid A V V' \mid \varepsilon
 \end{aligned}$$

Fattorizziamo

$$\begin{aligned}
 S &\rightarrow a A c S \mid c \\
 A &\rightarrow c P e A' \\
 A' &\rightarrow e c A' \mid \varepsilon \\
 P &\rightarrow c P' \\
 P' &\rightarrow X \mid H c \\
 H &\rightarrow H' \\
 H' &\rightarrow a H' \mid \varepsilon \\
 R &\rightarrow k C k \mid h C \\
 X &\rightarrow c a X' \\
 X' &\rightarrow c X' \mid \varepsilon \\
 C &\rightarrow i A V \mid R \\
 V &\rightarrow V' \\
 V' &\rightarrow e V' \mid A V V' \mid \varepsilon
 \end{aligned}$$

$$\begin{aligned}
 \text{FIRST}(S) &= \{ a, c \} \\
 \text{FIRST}(A) &= \{ c \} \\
 \text{FIRST}(A') &= \{ e, \varepsilon \} \\
 \text{FIRST}(P) &= \{ c \} \\
 \text{FIRST}(P') &= \text{FIRST}(X) \cup \text{FIRST}(H) = \{ a, c \} \\
 \text{FIRST}(H) &= \text{FIRST}(H') = \{ a, \varepsilon \} \\
 \text{FIRST}(H') &= \{ a, \varepsilon \} \\
 \text{FIRST}(R) &= \{ k, h \} \\
 \text{FIRST}(X) &= \{ c \} \\
 \text{FIRST}(X') &= \{ c, \varepsilon \} \\
 \text{FIRST}(C) &= \{ i \} \cup \text{FIRST}(R) = \{ i, h, k \} \\
 \text{FIRST}(V) &= \text{FIRST}(V') = \{ e, c, \varepsilon \} \\
 \text{FIRST}(V') &= \{ e, \varepsilon \} \cup \text{FIRST}(A) = \{ e, c, \varepsilon \}
 \end{aligned}$$

$\text{FOLLOW}(S) = \{ \$ \}$
 $\text{FOLLOW}(A) = \{ c \} \cup \text{FIRST}(V) = \{ c, e \} \cup \text{FOLLOW}(C) \cup \text{FIRST}(V') = \{ c, e \} \cup \text{FOLLOW}(C) \cup \text{FOLLOW}(V') = \{ c, e, k \}$
 $\text{FOLLOW}(A') = \text{FOLLOW}(A) = \{ c, e, k \}$
 $\text{FOLLOW}(P) = \{ e \}$
 $\text{FOLLOW}(P') = \text{FOLLOW}(P) = \{ e \}$
 $\text{FOLLOW}(H) = \{ c \}$
 $\text{FOLLOW}(H') = \text{FOLLOW}(H) = \{ c \}$
 $\text{FOLLOW}(R) = \text{FOLLOW}(C) = \{ k \}$
 $\text{FOLLOW}(X) = \text{FOLLOW}(P') = \{ e \}$
 $\text{FOLLOW}(X') = \text{FOLLOW}(X) = \{ e \}$
 $\text{FOLLOW}(C) = \{ k \} \cup \text{FOLLOW}(R) = \{ k \}$
 $\text{FOLLOW}(V) = \text{FOLLOW}(C) \cup \text{FIRST}(V') = \{ k \} \cup \text{FOLLOW}(V') = \{ k, e, c \}$
 $\text{FOLLOW}(V') = \text{FOLLOW}(V) = \{ k, e, c \}$

Calcoliamo gli insiemi guida delle produzioni

$\text{IG}(S \rightarrow a A c S) = \{ a \}$
 $\text{IG}(S \rightarrow c) = \{ c \}$
 $\text{IG}(S \rightarrow a A c S) = \{ a \}$
 $\text{IG}(A \rightarrow c P e A') = \{ c \}$
 $\text{IG}(A' \rightarrow e c A') = \{ e \}$
 $\text{IG}(A' \rightarrow \varepsilon) = \text{FOLLOWS}(A') = \{ c, e, k \}$
 $\text{IG}(P \rightarrow c P') = \{ c \}$
 $\text{IG}(P' \rightarrow X) = \{ c \}$
 $\text{IG}(P' \rightarrow Hc) = \{ a, c \}$
 $\text{IG}(H \rightarrow H') = \{ a, c \}$
 $\text{IG}(H' \rightarrow a H') = \{ a \}$
 $\text{IG}(H' \rightarrow \varepsilon) = \{ c \}$
 $\text{IG}(R \rightarrow k C k) = \{ k \}$
 $\text{IG}(R \rightarrow h C) = \{ h \}$
 $\text{IG}(X \rightarrow c a X') = \{ c \}$
 $\text{IG}(X' \rightarrow c X') = \{ c \}$
 $\text{IG}(X' \rightarrow \varepsilon) = \{ e \}$
 $\text{IG}(C \rightarrow i A V) = \{ i \}$
 $\text{IG}(C \rightarrow R) = \{ k, h \}$
 $\text{IG}(V \rightarrow V') = \{ e, c, k \}$
 $\text{IG}(V' \rightarrow e V') = \{ e \}$
 $\text{IG}(V' \rightarrow A V V') = \{ c \}$
 $\text{IG}(V' \rightarrow \varepsilon) = \{ k, e, c \}$

La grammatica non è LL(1) poichè se consideriamo gli insiemi guida delle produzioni

$\text{IG}(A' \rightarrow e c A') = \{ e \}$
 $\text{IG}(A' \rightarrow \varepsilon) = \text{FOLLOWS}(A') = \{ c, e, k \}$
 $\text{IG}(P' \rightarrow X) = \{ c \}$
 $\text{IG}(P' \rightarrow Hc) = \{ a, c \}$

$$IG(V' \rightarrow eV') = \{ e \}$$

$$IG(V' \rightarrow \varepsilon) = \{ k, e \}$$

$$IG(V' \rightarrow AVV') = \{ c \}$$

$$IG(V' \rightarrow \varepsilon) = \{ e, c, k \}$$

esse non hanno intersezione vuota

	a	C	e	i	k	h	\$
S	$\rightarrow a A c S$	$\rightarrow c$					
A		$\rightarrow c P e A'$					
A'		$\rightarrow \varepsilon$	$\rightarrow e c A'$ $\rightarrow \varepsilon$		$\rightarrow \varepsilon$		
P		$\rightarrow c P'$					
P'	$\rightarrow H c$	$\rightarrow X$ $\rightarrow H c$					
H	$\rightarrow H'$	$\rightarrow H'$					
H'	$\rightarrow a H'$	$\rightarrow \varepsilon$					
R					$\rightarrow k C k$	$\rightarrow h C$	
X		$\rightarrow c a X'$					
X'		$\rightarrow c X'$	$\rightarrow \varepsilon$				
C				$\rightarrow i A V$	$\rightarrow R$	$\rightarrow R$	
V		$\rightarrow V'$	$\rightarrow V'$		$\rightarrow V'$		
V'		$\rightarrow A V V'$ $\rightarrow \varepsilon$	$\rightarrow e V'$ $\rightarrow \varepsilon$		$\rightarrow \varepsilon$		

3.

$$S \rightarrow h A b S | c$$

$$D \rightarrow g E | g H d$$

$$A \rightarrow b D D e | D f$$

$$E \rightarrow E t | w$$

$$B \rightarrow C k | h$$

$$H \rightarrow H w H | H H w | i$$

$$C \rightarrow i A D | B$$

$$Z \rightarrow D Z | E$$

Eliminiamo la ricorsione sinistra

$$S \rightarrow h A b S | c$$

$$D \rightarrow g E | g H d$$

$$A \rightarrow b D D e | D f$$

$$E \rightarrow w E'$$

$$E' \rightarrow t E' | \varepsilon$$

$$B \rightarrow h B' | i A D k B'$$

$$B' \rightarrow k B' | \varepsilon$$

$$H \rightarrow i H'$$

$$H' \rightarrow w H H' | H w H' | \varepsilon$$

$$Z \rightarrow D Z \mid E$$

Fattorizziamo

$$S \rightarrow h A b S \mid c$$

$$D \rightarrow g D'$$

$$D' \rightarrow E \mid H d$$

$$A \rightarrow b D D e \mid D f$$

$$E \rightarrow w E'$$

$$E' \rightarrow t E' \mid \varepsilon$$

$$B \rightarrow h B' \mid i A D k B'$$

$$B' \rightarrow k B' \mid \varepsilon$$

$$H \rightarrow i H'$$

$$H' \rightarrow w H H' \mid H w H' \mid \varepsilon$$

$$Z \rightarrow D Z \mid E$$

$$\text{FIRST}(S) = \{ h, c \}$$

$$\text{FIRST}(D) = \{ g \}$$

$$\text{FIRST}(D') = \text{FIRST}(E) \cup \text{FIRST}(H) = \{ w, i \}$$

$$\text{FIRST}(A) = \{ b \} \cup \text{FIRST}(D) = \{ b, g \}$$

$$\text{FIRST}(E) = \{ w \}$$

$$\text{FIRST}(E') = \{ t, \varepsilon \}$$

$$\text{FIRST}(B) = \{ h, i \}$$

$$\text{FIRST}(B') = \{ k, \varepsilon \}$$

$$\text{FIRST}(H) = \{ i \}$$

$$\text{FIRST}(H') = \{ w, \varepsilon \} \cup \text{FIRST}(H) = \{ w, i, \varepsilon \}$$

$$\text{FIRST}(Z) = \text{FIRST}(D) \cup \text{FIRST}(E) = \{ g, w \}$$

$$\text{FOLLOW}(S) = \{ \$ \}$$

$$\text{FOLLOW}(D) = \{ e, f, k \} \cup \text{FIRST}(D) \cup \text{FIRST}(Z) = \{ e, f, k, g, w \}$$

$$\text{FOLLOW}(D') = \text{FOLLOW}(D) = \{ e, f, k, g, w \}$$

$$\text{FOLLOW}(A) = \{ b \} \cup \text{FIRST}(D) = \{ b, g \}$$

$$\text{FOLLOW}(E) = \text{FOLLOW}(D') \cup \text{FOLLOW}(Z) = \{ e, f, k, g, w \}$$

$$\text{FOLLOW}(E') = \text{FOLLOW}(E) = \{ e, f, k, g, w \}$$

$$\text{FOLLOW}(B) = \{ \}$$

$$\text{FOLLOW}(B') = \text{FOLLOW}(B) = \{ \}$$

$$\text{FOLLOW}(H) = \{ d, w \} \cup \text{FIRST}(H') = \{ d, w, i \} \cup \text{FOLLOW}(H') = \{ d, w, i \}$$

$$\text{FOLLOW}(H') = \text{FOLLOW}(H) = \{ d, w, i \}$$

$$\text{FOLLOW}(Z) = \{ \}$$

Calcoliamo gli insiemi guida delle produzioni

$$\text{IG}(S \rightarrow h A b S) = \{ h \}$$

$$\text{IG}(S \rightarrow c) = \{ c \}$$

$$\text{IG}(D \rightarrow g D') = \{ g \}$$

$$\text{IG}(D' \rightarrow E) = \{ w \}$$

$$\text{IG}(D' \rightarrow H d) = \{ i \}$$

$$\text{IG}(A \rightarrow b D D e) = \{ b \}$$

$$\text{IG}(A \rightarrow D f) = \{ g \}$$

