Nontrivial Solutions of Nonlinear Functional and Differential Equations

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Abstract

This thesis is concerned with the problem of finding nontrivial solutions of nonlinear equations.

Chapter one is an introduction to the concepts used through the thesis, including the notion of topological degree, measure of noncompactness, fixed point index and so on.

The work of chapter two builds a new definition of spectrum for nonlinear, finitely continuous maps using the class of A-proper mappings. In this chapter we also investigate the properties of the new spectrum and we discuss advantages and disadvantages of such a finite-dimensional approach.

In chapter three, by using fixed point index theory, we establish new results for some three point boundary value problems (BVPs) that have been previously studied by various authors, for example by Gupta *et al.* For certain values of a parameter α these particular BVPs can generate a continuous kernel that changes sign, so that positive solutions may not exist. We obtain existence of at least one or of multiple nonzero solutions.

In chapter four we extend the results of chapter three, allowing more general functions f and discontinuous kernels. We focus on a particular BVP that leads precisely to this situation, obtaining again, under suitable conditions, existence of nonzero solutions.

Finally, in chapter five, we turn our attention to the problem of eigenvalues of some three point BVPs. By using some results of chapter three and four together with a well known theorem on eigenvalues, we prove the existence of positive (and negative) eigenvalues.