

An extended notion of Cellular Automata for surface flows modelling

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Abstract: - Cellular Automata (CA) represent an alternative approach to differential equations to model and simulate complex fluid dynamical systems, whose evolution depends on the local interactions of their constituent parts. A new notion of CA was developed by our research group *Empedocles*. It permitted to improve significantly an empirical method for modelling macroscopic phenomena, concerning surface flows. This approach was applied to lava flows, to debris flows and to pyroclastic flows. This paper presents the CA extended notion together with the improved empirical method. Examples of simulations are exhibited and compared with the real events in order to show the method efficiency.

Key-Words: - Cellular automata, Modelling, Simulation, Fluid-dynamics, Lava, Debris flows, Pyroclastic flows

1 Introduction

Fluid-dynamics is an important field of Cellular Automata (CA) applications: lattice gas automata models [8] were introduced for describing the motion and collision of “particles” on a grid. It was shown that such models can simulate fluid dynamical properties; the continuum limit of these models leads to the Navier-Stokes equations.

A different approach characterises the so-called lattice Boltzmann models [2], where the state variables can take continuous values, as they are supposed to represent the density of fluid particles, endowed with certain properties, located in each cell (here space and time are discrete, as in lattice gas models).

Both approaches don't permit to make velocity explicit: a fluid amount moves from a cell to another one in a CA step (which is a constant time), it implies a constant “velocity” in the CA context of discrete space/time. Nevertheless, velocities can be deduced by analyzing the global behavior of the system in time and space. In such models, the flow velocity can be deduced by averaging on the space (i.e. considering clusters of cells) or by averaging on the time (e.g. considering the average velocity of the advancing flow front in a sequence of CA steps).

Many complex macroscopic fluid dynamical phenomena, which own the same locality property of CA, the surface-flows, like lava flows or debris flows seem difficult to be modelled in these CA frames, because they take place on a large space scale and need practically a macroscopic level of description that involves the management of a large amount of data, e.g., the morphological data.

An empirical method was developed in order to overcome these limits [7]:

- a nearly unlimited number of states is permitted; the state is composed of substates, each substate describes a feature of the space portion related to the own cell (e.g. the substate “temperature”);
- the transition function is split in several parts, each one corresponds to an “elementary” process of the macroscopic phenomenon (e.g. lava solidification);
- substates of type “outflow” are used in order to account for quantities moving from a cell toward another one in the neighbouring.

This choice doesn't overcome the problem of the constant “velocity”. Furthermore the substates “outflows”, computed in the step n , are effective at step $n+1$ when the neighbourhood conditions are changed. It involves precision lacks, that could be dangerous.

In the new proposed solution, the quantities (which move determining flows, e.g., the lava) are characterised by the substates specifying the mass centre position and velocity. When a fluid amount is computed to pass to another cell, then, in the same step, this part is added to the cell, altering the corresponding substates, which specify the mass centre position and velocity.

This operation is forbidden in the CA context, but it is admissible in particular conditions; as shown in the next section. In fact, the second section considers the new CA methodological approach for modelling macroscopic surface flows; the third section presents the applications to lava, debris and pyroclastic flows; some conclusions are reported at the end.

2 The methodological approach

Classical CA [9] were one of the first Parallel Computing models; they are based on a regular division of the space in regular cells (cellular space), each one embedding an identical computational device: the finite automaton (*fa*), whose state accounts for the temporary features of the cell; S is the finite set of states. The *fa* input is given by the states of $m > 1$ neighbouring cells, including the cell embedding the *fa*.

The neighbourhood conditions are determined by a pattern, which is invariant in time and constant over the cells. The *fa* have an identical state transition function $t: S^m \rightarrow S$, which is simultaneously applied to each cell. At time $t=0$, *fa* are in arbitrary states and the CA evolves changing the state of all *fa* simultaneously at discrete times (the CA steps), according to the transition function of the *fa*.

Such a definition is not sufficient for modelling spatially extended natural macroscopic phenomena as surface flows; more specifications need for permitting a correspondence between the system with its evolution in the physical space/time, on the one hand, and the model with the simulations in the cellular space/time, on the other hand.

Furthermore the complexity of macroscopic natural phenomena demands an extension of the original computational paradigm for many cases. The following considerations about CA and macroscopic systems introduce an extended definition of CA.

2.1 Global parameters

Primarily, the dimension of the cell (e.g. specified by the cell side p_s) and the time correspondence to a CA step p_t (clock) must be fixed. These are defined as “global parameters”, as their values are equal for all the cellular space. They constitute the set P together with other global parameters, which are commonly necessary for simulation purposes.

2.2 Substates

The state of the cell must account for all the characteristics, relative to the space portion corresponding to the cell, which are assumed to be relevant to the evolution of the system. Each characteristic corresponds to a substate; permitted values for a substate must form a finite set.

When a characteristic (e.g. a physical quantity) is expressed as a continuous variable, a finite, but sufficient, number of significant digits are utilised, so that the set of permitted values is large but finite.

The substate value is considered always constant inside the cell (e.g. the substate altitude). Then the

cell size must be chosen small enough so that the approximation to consider a single value for all the cell extension may be adequate to the features of the phenomenon.

The set S of the possible values of state of a cell is given by the Cartesian product of the sets S_1, S_2, \dots, S_n of the values of substates: $S = S_1 \times S_2 \times \dots \times S_n$; the set Q is also defined: $Q = \{S_1, S_2, \dots, S_n\}$.

The cellular space should be three dimensional, but a reduction to two dimensions is allowed when quantities concerning the third dimension (the height) may be included among the substates of the cell in a phenomenon concerning the earth surface.

2.3 Elementary processes

The state transition function t must account for all the processes (physical, chemical, etc.), which are assumed to be relevant to the system evolution, which is specified in terms of changes in the states values of the CA space. As well as the state of the cell can be decomposed in substates, the transition function t may be split into “elementary” processes, defined by the functions s_1, s_2, \dots, s_p with p being the number of the elementary processes.

The elementary processes are applied sequentially according a defined order. Different elementary processes may involve different neighbourhoods; the CA neighbourhood is given by the union of all the neighbourhoods associated to each processes.

In the empirical approach of Di Gregorio and Serra [7], an elementary process is individuated by: $s: Q_a^m \rightarrow Q_b$, where Q_a and Q_b are Cartesian products of the elements of subsets of Q , m is the number of cells of the neighbourhood, involved in the elementary process; Q_a individuates the substates in the neighbourhood that effect the substate value change and Q_b individuate the cell substates that change their value. Furthermore the movement of a certain amount of fluid from a cell toward another cell is described introducing substates of type “outflows”, that specify the involved fluid quantities to be moved in the neighbourhood. Such quantities are computed at a determined step according on the neighbourhood states, but the outflows shift is effective only at the successive step (the outflows are added to the fluid of the cell and their new barycentre is computed), when neighbourhood conditions are different. That involves a time discrepancy, that introduces inaccuracy.

An expensive computational solution would be the extension of the neighbourhood in order to compute not only the own outflows, but also the outflows of the neighbourhood cells and select those outflows corresponding to own inflows. Then the values (barycentres and quantities) of outflows and inflows could be used properly without a step lag.

The same computational result is obtained by extending the definition of elementary process in the following way: $\mathbf{s}:Q_a^m @ Q_b^m$. This means that the computation of the outflow from cell i to cell j doesn't determine the new value of substate outflow (that doesn't exist more as substate), but it effects immediately the new value of substate "fluid" in the cell i by subtraction and the new value of substate "fluid" in the cell j by addition of the same quantity. Barycentres and velocities are computed consequently.

This extension is forbidden in CA context, but it is here permissible because addition and subtraction are commutative operations. All commutative operations permit such an extension. The effect is exactly the same as the extension of the neighbourhood as previously specified.

2.4 External influences

Sometimes, a kind of input from the "external world" to the cells of the CA must be considered; it accounts for describing an external influence which cannot be described in terms of local rules (e.g. the lava alimentation at the vents) or for some kind of probabilistic approach to the phenomenon. Of course special and/or additional functions must be given for that type of cells.

2.5 Dimensions of the cell size and clock

The choice of the value of the parameters cell size and clock is dependent on the elementary processes. They could be inhomogeneous in space and/or time: the opportune dimension of a cell can vary for different elementary processes; furthermore very fast local interactions need a step corresponding to short times on the same cell size; the appropriate neighbourhoods for different local interactions could be different. An obvious solution to these problems is the following: the smallest dimension of a cell must be chosen among the permitted dimensions of all the local interactions. Then it is possible to define for each local interaction an appropriate range of time values in correspondence of a CA step; the shortest time necessary to the local interactions must correspond to a step of the CA. It is possible, when the cell dimension and the CA step are fixed, to assign an appropriate neighbourhood to each local interaction; the union of the neighbourhoods of all the local interactions must be adopted as the CA neighbourhood.

2.6 CA formal definition

Considering these premises, the following formal definition of two dimensional square or hexagonal CA for surface flows is given:

$$\langle \mathbf{R}, \mathbf{G}, \mathbf{S}, \mathbf{X}, \mathbf{P}, t, \mathbf{g} \rangle$$

- $\mathbf{R} = \{(x, y) | x, y \in \mathbb{Z}, -l_x \leq x \leq l_x, -l_y \leq y \leq l_y\}$ is the set of points with integer co-ordinates in the finite region, where the phenomenon evolves. Each point identifies a square or hexagonal cell.

- $\mathbf{G} = \{G_1, \dots, G_n\}$ is the set of cells, which undergo to the influences of the "external world"; n external influences are here considered, each one defines a subregion G_i of the cellular space, where the influence is active. Note that $G_i \cap G_j = \emptyset$.

- \mathbf{X} , the neighbourhood index, is a finite set of two-dimensional vectors, which defines the set $N(X, i)$ of neighbourhood cells of the cell $i = \langle i_x, i_y \rangle$ as follows: let $X = \{\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{m-1}\}$ with $m = \#X$, then $N(X, i) = \{i + \mathbf{x}_0, i + \mathbf{x}_1, \dots, i + \mathbf{x}_{m-1}\}$; \mathbf{x}_0 is always the null vector. Note that $X = \{X_j \hat{\mathbf{e}}_j | j = 1, \dots, p\}$, where $X_j \hat{\mathbf{e}}_j$ is correlated to the j -th elementary process.

- $\mathbf{S} = S_1, S_2, \dots, S_s$ is the set of the state values; it is specified by the Cartesian product of the finite sets of the values of the s substates S_1, S_2, \dots, S_s .

- \mathbf{P} is the finite set of global parameters, which effect the transition function.

- $t: S^m @ S^m$ is the deterministic state transition. It is specified by the ordered set of elementary processes in order of application: $\langle \mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_p \rangle$.

- $\mathbf{g}: N \times G @ S$ expresses the external influences to cells of G in the cellular space; it determines the variation of the state S for the cells in G . N , the set of natural numbers, is here referred to the steps of CA. \mathbf{g} is specified by the sequential applications of the n functions $\mathbf{g}_j: N \times G_j @ Q_j$, $\mathbf{g}_2: N \times G_2 @ Q_2$, $\mathbf{g}_n: N \times G_n @ Q_n$ where Q_1, Q_2, \dots, Q_n are Cartesian products of the elements of subsets of Q ($Q = \{S_1, S_2, \dots, S_s\}$).

2.7 Velocity determination

The following three equations (deduced in sequence and similar to the Stokes equations) are adopted in order to determine the velocities of fluid quantity between two cells: F is the force, m is the mass of the fluid inside the cell, v is its velocity, t is the time, v_0 is the initial velocity, \mathbf{q} is the angle of the slope between the two cells, α is the friction parameter.

The equations describe a motion, which is depending on the gravity force and is opposed by friction forces. An asymptotic velocity limit is considered because the effect of the friction forces increases as the velocity increases.

$$F = mg \sin \mathbf{q} - \mathbf{a} m v \quad (1)$$

$$dv/dt = g \sin \mathbf{q} - \mathbf{a} v \quad (2)$$

$$v = (v_0 - g \sin \mathbf{q} / \mathbf{a}) e^{-\mathbf{a}t} + (g \sin \mathbf{q} / \mathbf{a}) \quad (3)$$

This improved approach for modelling surface flows by CA was applied to the models of lava flows (SCIARA, version γ), debris flows (SCIDDICA, version $S_{3\text{-hex}}$) and pyroclastic flows (PYR2).

3 Applications

Three very different phenomena, which were modelled by our research group *Empedocles* according the empirical method of [7], are considered: lava flows [1, 3], debris flows [5, 6] and pyroclastic flows [4]. The models were partially or totally modified according the indications of the methodological approach of the previous section and are illustrated in the following subsections.

3.1 SCIARA

The extended CA notion was applied to the version γ of **SCIARA**, *Simulation by Cellular Interactive Automata of the Rheology of Aetnean lava flows* (sciara means the solidified lava path in Sicilian).

3.1.1 The model SCIARA- γ

The version γ of SCIARA is the septuple:

$$\text{SCIARA-}\gamma = \langle R, V, X, S, P, t, g \rangle$$

- $R = \{(x, y) \mid x, y \in \mathbb{Z}^2, -l_x \leq x \leq l_x, -l_y \leq y \leq l_y\}$ identifies the set of regular hexagons covering the finite region, where the phenomenon evolves.
- V is the set of cells, corresponding to the vents.
- $X = \{(0,0), (0,1), (0,-1), (1,0), (-1,0), (1,1), (-1,-1)\}$.
- S is the set of *fa* states, i.e. the Cartesian product of the values sets of the following substates:
 - S_a is the cell altitude;
 - S_{th} is the thickness of lava inside the cell;
 - S_x, S_y are the co-ordinates of mass centre of lava inside the cell;
 - S_T is the lava temperature.
- P is the set of the global parameters:
 - p_a is the apothem of the cell;
 - p_{step} is the time correspondence of a step;
 - p_{TV} is the lava temperature at the vent;
 - p_{TS} is the lava solidification temperature;
 - p_{adhV} is the adherence (i.e. unmovable thickness of lava at the emission temperature) at the vents;
 - p_{adhS} is the adherence value at the solidification temperature;
 - p_{cool} is the cooling parameter;
 - p_{vj} is the “limit of velocity” for lava flows.
- $t: S^7 \rightarrow S^7$ is the deterministic transition function, composed by the following “elementary” processes:
 - determination of the lava flows by application of minimisation algorithm [7];
 - determination of the lava flows shift by application of velocity formulae;
 - mixing of inflows and remaining lava inside the cell (determines new thickness and temperature);
 - lava cooling by radiation effect and solidification;
- $g: Q_{th} \times N \times Q_{th} \times Q_T$ specifies the emitted lava from the source cells (vents) at the CA step $t \in \mathbb{N}$.

3.1.2 Simulations with SCIARA- γ

A first application of SCIARA- γ concerns the crisis in the autumn of 2002 at Mount Etna (Sicily). The eruption started October 24 on the NE flank of the volcano, with lava generated by a fracture between 2500 m a.s.l and 2350 m a.s.l., pointing towards the town of Linguaglossa. After 8 days, the flow rate diminished drastically, stopping the lava front towards the inhabited areas. The Fig.1 shows the real lava flow at the maximum extension, Fig. 2 shows the corresponding simulation. Comparison between real and simulated event is satisfying, if we compare involved areas, temperatures and lava thicknesses.

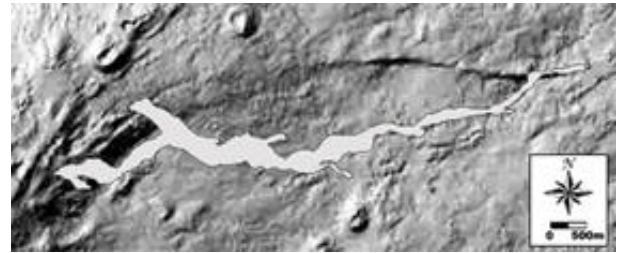


Fig.1. The 2002 Etnean lava flow of NE flank



Fig.2. 2002 Etnean lava flow simulation (NE flank)

3.2 SCIDDICA

The extended CA notion (but not the velocity formulae) was applied to the version $S_{3\text{-hexg}}$ of **SCIDDICA**, *Simulation through Computational Innovative methods for the Detection of Debris flow path using Interactive Cellular Automata* (sciddica means “it slides” in Sicilian). SCIDDICA

3.2.1 The model SCIDDICA $S_{3\text{-hexg}}$

The version $S_{3\text{-hexg}}$ of SCIDDICA is the quintuple:

$$\text{SCIDDICA } S_{3\text{-hexg}} = \langle R, X, S, P, t \rangle$$

- $R = \{(x, y) \mid x, y \in \mathbb{Z}^2, -l_x \leq x \leq l_x, -l_y \leq y \leq l_y\}$ identifies the set of regular hexagons covering the finite region, where the phenomenon evolves.
- $X = \{(0,0), (0,1), (0,-1), (1,0), (-1,0), (1,1), (-1,-1)\}$
- S is the set of *fa* states, i.e. the Cartesian product of the values sets of the following substates:
 - S_a is the cell altitude
 - S_{th} is the thickness of debris inside the cell
 - S_e is the energy of landslide debris
 - S_d is the depth of erodable soil cover

- \mathbf{P} is the set of the global parameters:
 - p_a is the apothem of the cell;
 - p_{step} is the time correspondence of a step;
 - p_{fa} is the height threshold (related to friction angle) for debris flows;
 - p_{rl} is the run-up loss (at each step), due to frictional effects;
 - p_{adh} is the adhesion (i.e. the unmovable amount of landslide debris);
 - p_{mt} is the activation threshold for mobilisation of the soil cover;
 - p_{pe} is the parameter of progressive erosion of the soil cover;
- $t:S^7 @ S^7$ is the deterministic transition function, composed by the following “elementary” processes:
 - debris flows determination by application of minimisation algorithm [7] with run up [5,6];
 - mixing of inflows and remaining debris inside the cell (determines new thickness and energy);
 - mobilisation triggering and effect;
 - energy loss by friction.

3.2.2 Simulations with SCIDDICA S_{3-hexy}

SCIDDICA S_{3-hexy} was applied to the Chiappe di Sarno (Italy) debris flows, triggered on 5-6 May 1998 by heavy rains. Debris slides were originated in the soil mantle, and transformed into rapid/extremely rapid debris flows, deeply eroding the soil cover along their path with an avalanche effect. Landslides caused serious damage and numerous victims.

Fig.3 shows the superposition of real and simulated events: light grey = only real debris flows, dark grey = only simulated flows, black = both real and simulated debris flows. Comparison between real and simulated event is satisfying, if we compare the involved area and debris thickness.

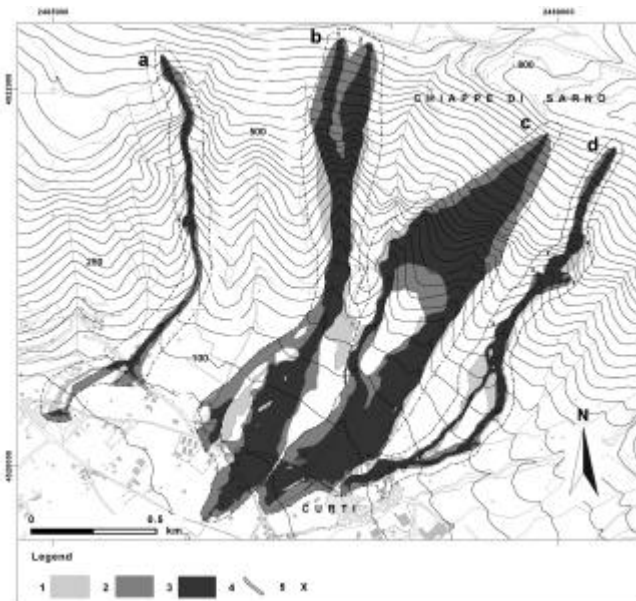


Fig.3. Superposition of real and simulated landslides

3.3 PYR

The extended CA notion was applied to PYR, generating the model PYR2 for pyroclastic flows [4].

3.3.1 The model PYR2

The model PYR2 is the septuplet:

$$\text{PYR2} = \langle \mathbf{R}, \mathbf{G}, \mathbf{X}, \mathbf{S}, \mathbf{P}, t, g \rangle$$

- $\mathbf{R} = \{(x, y) | x, y \in \mathbb{Z}^2, -l_x \leq x \leq l_x, -l_y \leq y \leq l_y\}$ identifies the set of squares covering the finite region, where the phenomenon evolves.
- \mathbf{G} is the set of cells, corresponding to the area, where the volcanic column begins to collapse and to generate the pyroclastic flows.
- $\mathbf{X} = \{(0,0), (0,1), (0,-1), (1,0), (-1,0)\}$.
- \mathbf{S} is the set of fa states, i.e. the Cartesian product of the values sets of the following substates:
 - S_a is the cell altitude;
 - S_{th} is the thickness of solid particles deposit;
 - S_x, S_y, S_z are the co-ordinates of the mass centre of the pyroclastic column inside the cell;
 - S_E is the elevation of the pyroclastic column;
 - S_V is velocity of the pyroclastic column;
 - S_{RT} is the residual time of the CA step during which a flow may leave the cell.
- \mathbf{P} is the set of the global parameters:
 - p_s is the side of the cell;
 - p_{step} is the time correspondence of a step;
 - p_{SP}, p_G is the solid particles and gas content of pyroclastic column (in percent: $p_{SP} + p_G = 100$);
 - p_{erl} is the degassing - particles deposition relaxation rate (elevation loss rate);
 - p_a is a parameter ruling the friction effect;
- $t:S^7 @ S^7$ is the deterministic transition function, composed by the following “elementary” processes:
 - degassing and particles deposition;
 - internal shift of the pyroclastic column;
 - flows determination and composition in the cell;
- $g:S_E \times \mathbb{N} @ S_E$ specifies the feeding of pyroclastic matter, $t \in \mathbb{N}$, which is the natural numbers set

3.3.2 Simulations with PYR2

PYR2 was applied to a pyroclastic flows which occurred in Montserrat, a Caribbean Island south east from Puerto Rico. They were generated on 12 May 1996 by the Soufriere Hills volcano in Montserrat. The eruption started in the morning, and after 3 hours of intermittent rockfalls, part of the north-eastern flank of the growing dome at Soufriere Hills Volcano collapsed producing a pyroclastic flow which reached the sea nearly 3km away.

The simulation results seems satisfying enough, if the comparison between real (Fig.4) and simulated (Fig.5) event is performed, considering the pyroclastic flow path and the area involved in the event.

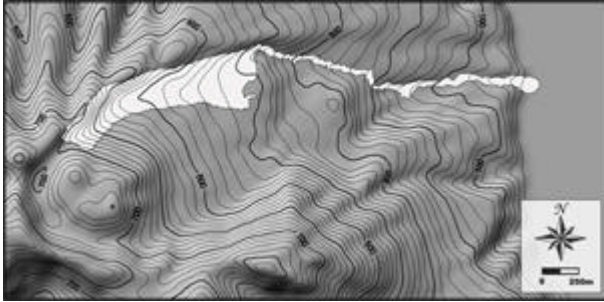


Fig.4. The May 1996 Soufriere Hills pyroclastic flow

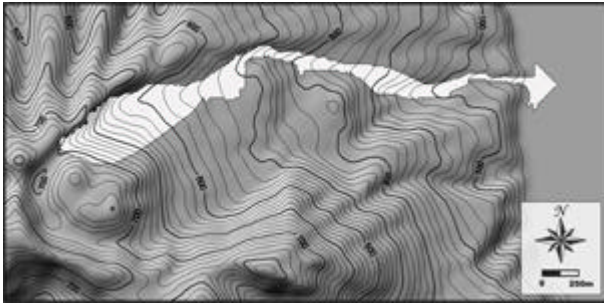


Fig.5. Simulation of the May 1996 Soufriere Hills pyroclastic flow

4 Conclusions

Our interdisciplinary research group *Empedocles* developed a new empirical approach for modelling and simulating complex macroscopic phenomena with CA and is applying such method to problems that are very difficult to be managed with differential equation systems.

The a-centric *Weltanschauung* (world-view) which characterises CA models involves a different viewpoint, with respect to partial differential equations, in treating complex macroscopic phenomena. Therefore, physics laws of conservation have to be rewritten (at a given approximation level) in a very different context of space-time discretisation. Values of model parameters are very important, (little changes produce very different simulations, that is typical of non-linear systems), they cannot always be determined directly, e.g. by physical measures: they are commonly selected, in an iterative way, by comparing the results of simulations with the global behaviour of the real phenomenon (genetic algorithms are a very useful instrument). These values must be considered only as the “optimal combination” of such parameters, which allows the model to better simulate the considered phenomenon.

The range of applicability of the CA model strongly depends on its framework, in terms of elementary processes, substates and global parameters, which refer to the physical characteristics of the real phenomenon to be

simulated. First, such a range can only be hypothesised, on the base of the characteristics of the local empirical laws considered in the transition function. The definitive judgment on the validity of the model depends on the comparison between a large set of simulations and the real phenomenon, but, when the model is validated, it may be used effectively for forecasting purpose in similar cases.

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